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CURRICULUM AND TEACHING OF MATHEMATICS IN THE HIGHER SECONDARY SCHOOLS.

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This curriculum guide for general mathematics consists of the development of a number of basic concepts which are chosen because of their relevance to problems of applications. These concepts are classified under the following strands--(1) concept of number, (2) concepts basic to operations, (3) concepts of percent and percentage, (4) concepts basic to geometric form and position, (5) concepts of measurement, (6) concepts of functional relationship, (7) concepts of comparison, (8) concepts of probability, (9) concepts of set, (10) concepts of limit, (11) concepts of infinity. All the concepts are developed continuously and simultaneously through four years of high school and at varying leves of sophistication and difficulty. (RP)

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NIE-HEW PROJECT NO.009
CURRICULUM AND TEACHING OF MATHEMATICS
IN THE
HIGHER SECONDARY SCHOOLS

(Curriculum-Guide in General Mathematics
for Higher Secondary Classes)

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NIE-HEW PROJECT 009.

Curriculum - Guide
in
General Mathematics
for
Higher Secondary Classes.

DEPARTMENT OF CURRICULUM AND EVALUATION
(Erstwhile Department of Curriculum, Methods and Textbooks)
(National Council of Educational Research & Training)

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NEW DELHI.

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BASIC ASSUMPTIONS

The curriculum in mathamatics should contribute as directly as possible to the achievement of avowed national goals of education. Some statements of the basic assumptions about the forces and conditions that are implicit in the educational programme of our country, are, therefore, in order,

We believe

- In creating a socialistic pattern of society in which every citizen will have an equality of opportunity to develop himself/herself to his/her best.
- in democracy and in fostering values associated with democracy as a way of life.
- that the trend towards rapid industrialisation will continue and the pace will be further speeded up with the passage of time.
- that a certain amount of scientific and technical knowledge is necessary for everyone in today's world.
- that there will be greater emphasis on scientific methods of thinking and on solving problems of our society from the scientific point of view.
- that there is a rapid and continuous expansion of the frontiers of knowledge in all the areas.
- in safeguarding our freedom and achieving self-sufficiency in the matter of food and other essentials.

PART I.

POINTS OF VIEW, OBJECTIVES AND CONTENT OF THE PROGRAMME

Section 1.

Concept of General Mathematics

The Objectives

Section 2.

The Content of the programme.

Section I.

THE PLACE OF GENERAL MATHEMATICS IN THE EDUCATIONAL PROGRAMME

Meaning of General Mathematics

The foregoing assumptions clearly indicate the dual need for mathematics. Firstly, it is an essential factor in the preparation of leaders for science, technology, industry, agriculture and other major areas of modern life. Secondly, it has a much bigger role to play as a part of the equipment of the ordinary citizen. There will be a substantial number of pupils who will not go up for any university or professional course after the secondary schools. These pupils should be provided the needed education to live more effectively as responsible and intelligent members of the society. In any programme of general education of these pupils, mathematics is indispensible for it has important contributions to make.

The term General Mathematics is used here for the programme in mathematics designed to further the needs of general education which aims at the fullest realization of individual's potentialities and the development of common values, attitudes, understandings and skills needed by all for becoming intelligent citizens. It is difficult to give a precise definition of such a programme. A few factors which explain the nature of General Mathematics for the purpose of this guide are listed here:

(1) Our present day society makes increasing demands on the mathematical understanding of the citizen.

General Mathematics should, therefore, acquaint the pupils with those mathematical concepts and methods which will enable them to solve problems of quantitative and spatial relationships. It should also acquaint the pupils with practices and institutions using mathematics in furthering the advance of human knowledge or in solving personal and social problems.

(2) The use of mathematics in all the fields of knowledge has rapidly increased. Even in the social sciences, which deal with persons in their social, economic and psychological relationships we find an increasing use of the ideas and notions of mathematics. Modern research in economics, business, education, industry and government is not possible without the use of the language and the methods of mathematics. The general mathematics course should enable the pupils recognize the relationships and uses of mathematics to other fields of knowledge.

(3) A great majority of pupils would become active procedures and consumers immediately after leaving school. General mathematics should include study of the applications of mathematics in personal, family and community life.

(4) General mathematics should not restrict itself to serving utilitarian purposes. It should give the pupils a taste of the nature and scope of mathematics as science and as a language of universal usage. Thus, it has to give special attention to the use of symbolic language in expressing thought in a concise form and in analysing problems.

(5) General mathematics should contribute to the development of the powers of independent, critical thinking and those attitudes of mind which are essential for intellegent citizenship in a democracy.

The Objectives of the Mathematics Programme

The General Mathematics programme presented in this Guide has been guided by the specific objectives listed below. Every child, to the limit of his capacity, should have the opportunity:

- To acquire knowledge of basic concepts, terms, principles, processes and relationships in Mathematics.
- To develop skill in mathematical computations.
- To develop skill in using mathematical instruments.
- To develop an ability to translate mathematical data given in one form into another form. (e.g., from descriptive form to symbolic form, from symbolic form to graphical form, etc.)
- To develop an ability to apply mathematical knowledge and skills in solving problems as they arise in life.
- To develop logical thinking and reesoning.
- To develop an ability to locate valid, relaible and relevant information in Mathematics.
- To develop interest in
  (a) reading mathematical material,
  (b) mathematical computations, and
  (c) other mathematical activities.
- To develop the following qualities:
  (a) Accuracy and precision
  (b) An attitude of checking computations.
  (c) Neatness in mathematical work

(d) Systematic presentation of arguments and computation.

(e) Power of observation, abstraction and generalisation.

(f) Doing mental calculations with speed.

(g) Making sound estimates.

— To develop appreciation of the language and history of Mathematics and its contribution to life and to the development of the other subjects.

The attainment of these objectives is possible

A By helping pupils to discover the following Basic Understandings:

(1) Mathematics is a collection of mathematical systems. Some of these systems, while logically independent of each other are nevertheless quite intimately related.

(2) A mathematical system contains

(a) undefined elements

(b) the postulates - the initial statements about elements, which are agreed to rather than proved by reasoning.

(c) a method of reasoning

(d) established conclusions or theorems.

(3) Mathematical concepts are generalisations or abstractions suggested by elements of familiar objects oe experiences.

(4) Mathematics as a method can be applied to any field. It is being adopted more and more by the physical sciences, the social sciences and philosophy.

(5) Discovery in mathematics, calls for creative ability and imagination which must go far beyond the limits of visualisation.

(6) Mathematics uses all types of reasoning to suggest possible conclusions but no conclusion is accepted until it has been proved deductively.

(7) All sciences are tending more and more towards mathematical form.

(8) The historical development of mathematics forms a facinating story of man's effort to acquire and develop useful ideas about number and space.

(9) Mathematics is a language of quantity, size and order. It is also a language for exploring logical relations.

B by helping pupils to acquire proficiency in the following skills and Abilities:

(1) Skill in computing with numbers with speed and accura

(2) Ability to perform calculations mentally.

(3) Ability to estimate and check results.

(4) Ability to use and interpret graphs, simple statistics and tabular presentations of quantitative data.

(5) Ability to make sound judgements with regard to practical, quantitative problems.

(6) Ability to use mathematical tools.

(7) Ability to think critically, to draw inferences and to generalise.

(8) Ability to identify a problem, define it clearly and to plan a procedure for its solution.

(9) Ability to represent designs and spatial relations by drawings.

C by helping pupils develop the following Attitudes:

(1) To work independently.

(2) To base judgements upon objectively verifiable data

(3) To solve problems by numerical methods.

(4) To appreciate the contribution of mathematics to personal lives as well as to society as a whole.

(5) To enjoy statistical comparisons, economic estimates and mathematical formulations.

(6) To appreciate geometric form as it occurs in nature, art, industry and architecture.

Objective in Terms of Behaviour Changes

The specific objectives cited above lay emphasis upon helping the pupils to mature and acquire desirable behaviours, including skills in the use of mathematics. The following are a few of different kinds of changes in behaviour that require direct provision for its practice in active learning situations and in types of evaluation.

OBJECTIVE 1 To acquire knowledge of basic concepts, terms processes and relationships in mathematics.

Specifications

A The pupil recognises concepts, terms, etc.

B He recalls terms, concepts, etc.

C He defines or describes terms etc. whenever required.

D He illustrates concepts, terms etc.

E He discriminates between closely related concepts, processes and principles.

F He compares and contrasts terms, concepts, principles, etc.

G He detects errors in statements, definitions etc.

OBJECTIVE 2 To develop skill in mathematical computations

Specifications

A The pupil does the computation with reasonable speed.

B He does the computation orally (where possible).

C He uses short cuts to save the time and energy.

D He develops the habit of precision and exactness.

E He detects the errors in processing.

F He checks the work done in mathematics before proceeding further, preferably by an alternative method.

G He develops self-reliance in handling numerical problems.

H He uses mathematical tables and ready reckoners.

I He uses the slide rule.

OBJECTIVE 3 To develop skill in using mathematical instruments.

Specifications

A The pupil selects appropriate instruments for measurement and drawing.

B He expresses the measurement in appropriate units

C He estimates geometrical magnitudes.

D He recognises the degree of accuracy required in a measurement.

E He detects errors in the instruments, if any.

F He exhibits a sense of symmetry and proportion.

G He draws geometrical figures accurately.

H He draws geometrical figures neatly.

I He prepares charts, graphs and plans to a given scale.

J He draws figures with reasonable speed.

OBJECTIVE 4 To develop an ability to translate mathematical data given in one form into another form.

Specifications

A The pupil reads a graph.

B He interprets graphical data in simple language

C He identifies the scales used in graphs.

D He recognises situations in which a graphical approach of representation serves a useful purpose.

E He translates symbolic terms into simple language and vice versa.

F He selects the right kind of graph for representing given data and draws it.

G He prepares a table or a graph from a given narrative data

H He translates a given statement into a geometrical figure.

I He reads a geometrical figure and expresses its details verbally.

J He abreviates a given quantitative statement by algebraic symbols.

OBJECTIVE 5 To develop an ability to apply mathematical knowledge and skill in solving problems as they arise in life.

Specifications

A The pupil analyses the data and finds what is given and what is to be found.

B He reconstructs the problem in a familiar context.

C He finds the adequacy or otherwise of the data.

D He selects the relevant data.

E He selects the appropriate method

F He estimates the result.

G He verifies the solution.

H He detects errors, if any, in a given solution.

I He suggests alternative methods of solving a problem.

J He constructs problems involving a given concept.

K He solves problems with different methods.

L He shifts relevant data for reaching the required result.

M He uses correct units.

N He classifies given data in various categories.

OBJECTIVE 6 To develop logical thinking and reasoning.

Specifications

- A The pupil is logical in solving problems.
- B He distinguishes assumptions from inferences.
- C He distinguishes between valid and invalid inferences.
- D He recognises the limitations of logical inferences in practical situations.
- E He selects relevant assumptions for the purpose of drawing certain inferences.
- F He builds up a logical sequence of the given statements.
- G He detects logical fallacies.
- H He draws inference from a given hypothesis.

OBJECTIVE 7 To develop an ability to locate valid, reliable and relevant information in mathematics.

Specifications

- A The pupil refers books, text and reference, for getting more material for a particular topic.
- B He locates the mathematical material in journals, newspapers etc.
- C He refers to original sources of information by consulting appropriate sources.
- D He checks the validity of a given information by consulting appropriate sources.
- E He knows about different sources useful for obtaining mathematical information.
- F He checks the reliability of information.

OBJECTIVE 8 To develop interest in (a) reading mathematical material, (b) mathematical computations, and (c) other mathematical activities.

Specifications.

- A The pupil reads mathematical journals and other related material.
- B He enjoys mathematical amusements and puzzles.
- C He plays mathematical games
- D He participates in the activities of mathematics club.

E He undertakes computational work voluntarily.

F He prepares mathematical models, charts, graphs designs, etc. in his spare time.

OBJECTIVE 9 To develop the following qualities in pupils

A Accuracy and precision

B An attitude of checking computations

C Neatness in computations

D Systematic presentation of arguments and computation

E Power of observation, abstraction and generalisation

F Doing mental calculations with speed

G Making sound estimates

Specifications

A The pupil checks his work at every step.

B He is particular about neat work.

C His calculations are legible and systematic.

D He presents the work in a systematic way.

E He likes neat and systematic work.

F He tries various techniques of checking results.

G He generalises on the basis of empirical data

H He persists in his endeavours.

OBJECTIVE 10 To develop appreciation of the language and history of Mathematics, and its contribution to life and to the development of other subjects.

Specifications

A The pupil shows readiness to work out problems in mathematics.

B He devotes more time for mathematics

C He expresses his interest in mathematics

D He uses his knowledge of mathematics in other subjects

E He participates in mathematical activities

F He reads mathematical literature.

Section 2.

THE GENERAL MATHEMATICS PROGRAMME.

There are two mutually related and interdependent phases of mathematics, namely, its social phase and its technical phase. In a meaningful mathematics programme a balanced recognition of both phases is essential. It is believed that general mathematics, in order to be meaningful should be taught in such a way that the learner not only understands the essential concepts, principles, and processes but also sees how what he is learning functions in daily life.

Broadly speaking two approaches to organization and presentation of mathematical subject-matter are possible. Firstly, certain concrete problem situation or needs in various areas of life can be located and pertinent mathematical concepts and skills used in these situations may be indicated. Secondly, certain basic concepts of mathematics may be chosen and their applications in life may be indicated. For the purpose of this guide, the second approach has been accepted because it is only through this organization that the basic structure of the subject can be preserved. We believe that for both intelligent use and true appreciation of mathematics, it is essential that pupils should understand the basic concepts, perceive patterns and understand the properties of mathematical systems.

The programme outlined in this guide consists of the development of a number of basic concepts and their related skills. These concepts are classified under the following strands:

1. Concepts of number,
2. Concepts basic to operations.
3. Concept of percent and percentage.
4. Concepts basic to geometric form and position.

...

Contd...

5. Concepts of measurement.
6. Concepts of functional relationship.
7. Concepts of comparison.
8. Concept of probability.
9. Concept of set.
10. Concept of limit. (as and when required)
11. Concept of infinity. (as and when required)

The task of isolating and categorizing basic concepts is not easy and the strands listed above are chosen because they are the foundations of any course in mathematics. It should be understood that the order in which the concepts are listed is not significant. None of the concepts is less or more important than any other and it is not expected that no rigid sequence will be followed in teaching them. ✓All the concepts are developed contrinously and simultaneously though at varying level of difficulty. In a particular class one or more of them may receive special emphasis. Also, it is possible that all people may not completely agree on the grade placement of these concepts, yet few people will disagree with the worth of these concepts for secondary classes.

In the first list of concepts there were about 350 concepts which have been reduced to 72 concepts in the final list. These concepts are basic to the mathematical thinking and numerous other concepts that are not mentioned can be classified under one of the concepts listed.

Contd...

BASIC MATHEMATICAL CONCEPTS

	Concepts	Years			
		1	2	3	4
I.	Concept of Number				
II.	Concepts Basic to Operations				
	One-to-one correspondence	x	x	x	x
	Rational number	x	x	x	x
	Real number	x	x	x	x
	Order	x	x	x	x
	Base	x	x	x	x
	Exponent	x	x	x	x
	Expression	x	x	x	x
	Reciprocal	x	x	x	x
	Addition	x	x	x	x
	Subtraction	x	x	x	x
	Multiplication	x	x	x	x
	Division	x	x	x	x
	Commutativity	x	x	x	x
	Associativity	x	x	x	x
	Distributivity	x	x	x	x
	Closure	x	x	x	x
	Identity element	x	x	x	x
	Estimation	x	x	x	x
	Variable	x	x	x	x
	Constant	x	x	x	x
	Inverse	x	x	x	x
	Equality	x	x	x	x
	Irrational number		x	x	x
	Absolute value		x	x	x
	Binary operations		x	x	x
	Radicals		x	x	x

Contd...

Concepts	Years			
	1	2	3	4
Imaginary number			x	x
Logarithm			x	x
Factorial			x	x
Permutation			x	x
Combination			x	x
III. Concept of Percent and Percentage				
Percent	x	x	x	x
Percentage	x	x	x	x
IV. Concepts Basic to Geometric Form and Position				
Point	x	x	x	x
Lines	x	x	x	x
Angles	x	x	x	x
Planes	x	x	x	x
Plane figures	x	x	x	x
Solid figures	x	x	x	x
Perpendicularity	x	x	x	x
Parallelism	x	x	x	x
V. Concepts of Measurement				
Length	x	x	x	x
Area	x	x	x	x
Volume	x	x	x	x
Weight	x	x	x	x
Time	x	x	x	x
Money	x	x	x	x
Angular measure	x	x	x	x

Contd...

	Concepts	Years			
		1	2	3	4
VI.	**Concepts of Functional Relationship.**				
	Formula	x	x	x	x
	Graph	x	x	x	x
	Equation	x	x	x	x
	Ordered pair		x	x	x
	Trigonometric ratios			x	x
	Relation			x	x
	Series				x
	Variation				x
	Mapping				x
	Functions				x
VII.	**Concepts of Comparison**				
	Ratio and proportion	x	x	x	x
	Similarity	x	x	x	x
	Congruency	x	x	x	x
	Equivalency	x	x	x	x
	Symmetry	x	x	x	x
	Inequality	x	x	x	x
	Statistical averages		x	x	x
VIII.	**Concept of Probability**				
	Probability				x
IX.	**Concept of Set**				
	Set	x	x	x	x
X.	**Concept of Limit**				
	Limit			x	x
XI.	**Concept of Infinity**				
	Infinity	(Whenever needed)			

Contd....

The Content Chart

An outline of the proposed content for the General Mathematics programme is presented in the following chart. There is no uniformity in the classes in which general mathematics is taught in various states. The duration varies from a one-year course to a four-year course. The content for this guide has been, therefore, selected for four-years, corresponding to classes VIII, IX, X and XI in States which follow seven-year elementary school pattern.

The chart indicates, in brief, the expansion of each concept from each level to the next and helps in knowing what precedes and follows a particular stage. It also specifies the aspects of each concept that are assumed to be appropriate for the majority of pupils during a particular year of instruction.

The chart is intended only to provide a convinient overview of the content each year and of the total programme.

Contd...

THE CONTENT CHART

IST YEAR	2ND YEAR	3RD YEAR	4TH YEAR
I. CONCEPT OF NUMBER		II. CONCEPT BASIC TO OPERATIONS	
1. Rational Numbers Definition and representation; Comparison; Order on a number line; Operations and properties; Reducing rationals to lowest.	1. Real Numbers Meaning of irrational and real numbers; Order; Absolute value; Operations and properties.	1. Logarithmic Numbers Meaning; Basic principles; Common logarithms.	1. Progressions Definition; Arithmetic progression; Geometric progression.
2. Real Numbers Notion through completeness of the number line; Order.	2. Algebraic Expressions Negative exponents; Laws; Radicals and fractional exponents; Square roots; Rationalization.	2. Permutation and Combination Meaning and symbolism; Arranging letters and other articles in different orders.	2. Permutation and Combination Definitions; Fundamental principle of occurance of events; Permutation and combination of n things taken r at a time; Relation between np_r and n_{C_r} .
3. Algebraic Expressions Meaning; Notion of a variable; Types; Polynomials over integers and rationals; Exponents; Substitution; Operations with expressions; Arrangement of expressions in descending and ascending powers of a variable; Products and factorization.			3. Binomial Theorem Concept of mathematical induction; Binomial expansion for positive integral index and proof for it.

Contd.....

1ST YEAR	2ND YEAR	3RD YEAR	4TH YEAR
	III. CONCEPT OF PERCENT AND PERCENTAGE		
1. Percent and Percentage Percentage of increase or decrease; Application of percentage in profit and loss, interest and budgeting; Compound interest by the use of tables.	1. Percent and Percentage Application of percentage in Banking, Discounts, Stocks and shares; Insurance, Taxes and cooperative societies.		
	IV. CONCEPTS BASIC TO GEOMETRIC FORM AND POSITION		V. CONCEPT OF MEASUREMENT
1. Point and Line Number line; Rulers and units of measure; Coordinate system; Betweenness; Segments, rays and lines; Collinear points; Intersection of lines; Idea of parallel lines; Constructions of right bisectors and parallel lines; Dividing a segment in a number of congruent segments.	1. Point and Line Parallel and skew lines; Transversals; Theorems on parallel lines; Inequalities for segments; Finding a third, fourth and mean proportional between a number of segments; Slope of a line.	1. Point and Line Theorems on betweenness and seperation.	
2. Plane and Space Definitions; Coplanar points; Flatness of planes, Intersection of planes and lines and planes; Convex sets, half planes and edges; Separation.	2. Plane and Space Theorems on lines and planes.	2. Plane and Space Theorems on parrallel lines and planes and perpendicular lines and planes in space.	
	3. Angle and Triangle Measures of the angles of a traingle; Intercepts made by a transversal on parallel lines; Inequalities for angles; Theorems on inequalities in a single traingle; Theorems on distance between a point and a line; Altitude of a triangle; Pythagorus theorem; Construction of triangles.	3. Angle and Triangle Theorems on right triangle; Indirect measurement; Concurrency.	

Contd...

1st Year	2ND YEAR	3RD YEAR	4TH YEAR
3. **Angle and Triangle** Angles, their parts and types; Triangles and their parts; Interior and exterior of an angle and a triangle; Perpendicularity; Congruent angles; Constructions on angles, perpendiculars, and triangles; Dividing a segment internally and externally in a given ratio.			
4. **Polygon and Polygonal Region** Definition of quadrilateral and its parts; Types of quadrilateral; Constructions on quadrilaterals; Recognition meaning and construction of a polygon; Meaning of a regular polygon.	4. **Polygon and Polygonal Region** Definitions and properties; Theorems on a) triangles and quadrilaterals standing on the same base and between the same parallels. b) quadrilaterals and c) area of quadrilaterals; Drawing figures equal in area to certain other figures; construction of similar polygons.	4. **Polygon and Polygonal Region** Theorems on quadrilaterals and polygons containing more than four sides.	
5. **Circle** Meaning and recognition; Drawing circles; The meaning of π (pie); Using formulae for circumference and area of a circle in problems of everyday life.	5. **Circle** Definitions; Theorems and constructions on chords and angles.	5. **Circle** Tangency; Cyclic quadrilaterals; Circumference and area of circle; Lengths of arcs and areas of sectors; Circles in a coordinate plane; Constructions.	
6. **Solids** Meaning and recognition; Using formulae for surface area and volume of right regular prism, right circular cylinder, and right circular cone.	6. **Solids** Meaning and recognition of sphere and pyramids; Surface and lateral area of right circular cone, right circular cylinder, and right regular pyramids, surface area of a sphere, and volume of right regular pyramid and sphere by formula.	6. **Solids** Definitions; Geometry of the sphere; Theorems on solids.	

Contd...

1ST YEAR	2ND YEAR	3RD YEAR	4TH YEAR
VI. CONCEPT OF FUNCTIONAL RELATIONSHIP			
1. Equations and Inequalities Formulas as equation; Mathematical sentences and statements; Open sentences and their types; Replacement and replacement for set of an open sentence.	1. Equations and Inequalities Solution and solution set of an open sentence, Definition; Use of two variables; Properties; Equation of a line.	1. Equations and Inequalities Meaning of a quadratic equation and a quadratic inequality; Introducing imaginary numbers; Types of quadratic equations; Solution and graphs of quadratic equations and quadratic inequalities.	1. Functions and Graphs Definition of a function; Inverse; Notation and graph; Special classes of functions; Trigonometric functions and their graphs; Variations in the values of trigonometric functions; Exponential functions and their graphs; Logarithmic functions and their graphs; Exponential equations and their solution.
2. Graphs Meaning; Interpretation of pictographs; Making bar and line graphs; Making and interpretting distribution graphs.	2. Graphs Meaning of origin, axis and coordinate; Plotting the points; Drawing line graphs; Graph of the solution set of an open sentence; Graph of inequality; Solution of problems by graphs.	2. Trigonometric Ratios Definitions; Relationships; Trigonometric ratios of acute and obtuse angles. 3. Relations and Graphs Ordered pairs and relations; Domain range and graph of a relation; Inverse of a relation and its graphs; Compound sentences and their graphs.	2. Trigonometric Ratios Trigonometric ratios of compound and multiple angles; Solution of right triangle problems.

Contd...

VII. CONCEPT OF COMPARISON

1ST YEAR	2ND YEAR	3RD YEAR	4th YEAR
1. Ratio and Proportion Definition; Equivalent ratios; Meaning and properties of proportion.	1. Ratio and Proportion Proportional segments; Mean proportionals; Application of ratio and proportion.	1. Statistics Measures of dispersion, their properties and uses.	
2. Congruence, Similarity and Symmetry One-to-one correspondence and congruence; Congruent segments, angles and triangles and properties with verification; Meaning of symmetry; Types of symmetry; Congruency of symmetrical figures; Symmetry of a segment, an angle and a circle.	2. Congruence, Similarity and Symmetry Application of congurence Meaning of similarity; Theorems on similarity of triangles; Areas of similar triangles; Theorems on congruent triangles; Construction of congruent and similar triangles.		
	3. Statistics Definition; Basic concepts; Graphes of frequency distributions; Measures of central tendency, their properties and uses.		

Contd...

1ST YEAR	2ND YEAR	3RD YEAR	4TH YEAR
VIII. CONCEPT OF PROBABILITY			
			1. Probability Meaning; Mutually exclussive, dependent and independent events; Basic Theorems.

PART II.

METHODS AND MATERIALS

Section 3.

The pupil

Principles of Learning

Motivation

Section 4.

Techniques of teaching

Section 5.

Adaptation of Instruction
to individual differences

Section 6.

Evaluation

Section 7.

Correlation

Section 3.

Factors Affecting Instruction.

Pupil

Educational Objectives for children are realized in the growth and learning of the individual child. Each teacher has to ensure that all that goes on in the classroom promotes maximum child growth and development toward our objectives. Certain principles of child growth, listed by educators, psychologists and others are given here. These principles are important to all those who are interested in mathematics education.

1. The child responds as a whole to the situation which confronts him. His intellectual, emotional, physical and social development must be taken into account in planning experiences.
2. Growth and development are continuous. There are no sharp breaks or sudden changes in the pattern of growth; certain types of changes in development may become prominent at a particular stage.
3. No two children are alike. They differ in
   (a) rate of growth
   (b) the pattern of growth
   (c) the maximum level of attainment.
4. Growth is largely based upon the interaction of individual's heredity and environment.
5. Appropriate growth is a prerequisite to productive learning.

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It goes without saying that the teacher should recognize the newer conception of the learner and the way he learns. Anything that he does must be done in terms of its effects on the child's total personality and, reciprocally, that total personality must be taken into acount in planning the experiences. Some emphasis should be placed on the general characteristics of children with respect to the intellectual, physiological, social and emotional aspects of their personality at the perticular maturity level. The syllabus proposed in this guide has been designed for children between the ages 13 to 16 years. It is felt that the following description of the characteristics of children in general may prove helpful in planning the experiences.

MENTA-LLY	These children are gaining in their intellectual ability and are ready to acquire added knowledge, skills and understanding.

They develop ability to reason, to draw on meanings, to make decisions, to evaluate actions and the like.

They grasp the meaning of abstract words.

PHYSICALLY	These children show marked development in body size and are gaining full maturity. They are developing in motor ability and are active, daring and energetic.
EMOTIONALLY	These children are often unstable and easily excited or depressed.

They show increasing awareness of self and self-ideals.
They resist domination and seek independence.

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SOCIALLY These children desire to gain status in social groups such as in the home and school.

They acquire new friends and interests.
They are cooperative and like to work as a team.
They have strong group loyalties.
They are interested in organized competitions.

Althought all of the above principles of growth and characteristics of pupils are important, the recognition of individual differences and the provision for these differences continues to be the most challenging problem in the teaching of mathematics.

Basic principles of learning

It is obviously true that the teachers of mathematics, if they are to fulfil their obligations of guiding the growth of their pupils toward desirable goals, must follow established principles of learning. Learning is defined as a change in behaviour brought about as a result of interaction of the child and his environment. Here, we are concerned with all desired changes in behaviour with reference to numerical, spatial, quantitative, and logical situations as have been accepted in the objectives. These changes in behaviour not only enable the child to acquire new knowledge or skill but also develop desirable attitudes and appreciations. These can be broadly classified as those related to:

1. Sensory - motor skills - use of tools, measuring and recording devices; making models, charts, graphs, etc.; memorizing tables and formulas; etc.

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2. Concepts, meanings and generalizations - defining terms; establishing relationships between facts and principles; selecting correct procedures; using correct language;
3. Motives, interests and attitudes - perticipating in mathematical games, contests, puzzles etc; reading books, magazines, stories; listening to radio programmes studying topics from the history of mathematics; etc.
4. problem-solving - obtaining and organizing relevant facts; drawing inferences; applying appropriate processes; checking results; etc.

These types of behaviours are not mutually exclusive and occur concurrently in any given mathematical situation, the degree of each depending upon the nature of the situation. In all cases in mathematics there is an attempt to raise the quality of behaviour from behaviours involving sensory motor skills to those which are problem solving in character. The child's ability to progress in mathematics is determined by his power to generalize, abstract, organize logically and relate those to more situations. Learning which is a developmental process is characterized by ability to think and to solve problems so far as learning mathematics is concerned.

Principles of learning have grown out of the study of two broad categories of theories viz., association and field. Each of these theories offer an explanation of the way the child learns. By studying these theories and drawing interpretations of these theories for learning of mathematics, the following important principles have been derived:

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a) Learning is continuous and developmental

The learning of mathematics follows sequential development Mathematics itself is sequential in nature. It starts from simple ideas and proceeds to more refined or more generalized ideas. The concepts of mathematics expand fan like into more and more refined and often more abstract concepts. Also the learning proceeds from a concrete first-hand situation to an abstract mathematical idea. The specific experiences and learning are not treated as self bounded units, each one complete and independent but are continuously reorganized to promote more mature form of behaviour. This reorganization and maturation develops ability to make computation with accuracy and speed, to apply principles with efficiency and to solve problems in an organized manner.

b) Learning is determined by the degree of maturation of the learner.

What a child can learn effectively depends very much on his maturity level. Maturation in terms of bodily growth is purely mechanical and there seems to be nothing much a teacher can do about it. HOwever it can be taken as a process directed toward and shaped up by a goal which becomes clearer and clearer. In mathematics, as a child strives to evolve a clear insight into the principle, proof or operation, he draws upon his own maturity of development and mental capacity and in turn acquires a higher level of maturity which brings him closer and closer to his objective. With increasing maturity the child gets a sort of mastery of what he has already learnt and is in a state of readiness for further work.

-: :-

c) Learning depends upon motivation

For successful teaching it is absolutely essential to arouse and mobilize the desire to learn. The learning should be organized in such a way as to get a grip on the motives and impulses of the learner. There is no simple method for stimulation pupils to learn. The desire to learn may arise from one or more of a variety of sources. The main motivating factors are clarity of goals, needs, social usefulness, successful performance, promotions, teacher personality, etc. Negative factors as failures, or punishment also have an effect. Since successful learning stimulates most children to further learning, every child should be made aware of his progress.

d) Learning requires active participation of the learner.

The learning takes place only by reacting. It is not a passive process of absorption. The child learns through participation in learning activities. Unfortunately mere recitation and memorization of facts are common in mathematics class-rooms. These are less likely to stimulate active participation. Much more likly to involve students actively in learning mathematics is a teaching procedure which encourages activities such as doing, comparing, contrasting, recalling facts learnt previously, making applications, reasoning, associating, and making generalizations. Since the learner is more likely to understand and remember those principles which he discovers, increased emphasis should be placed on his own experimentation and independent thinking.

e) Learning of skills or abilities depends on practice

Drill or practice is significant in learning mathematics. Practice is needed to perfect skills. It should, however, follow when the understanding of the process has been developed. It should be suitably spaced.

f) Learning by wholes is generally better than learning by parts

Effective understanding does not arise from the memorization of isolated facts or the mastery of a large assemblage of material. The learning should be organized in a way that the foremost interrelationship or unity of the facts is clearly recognized. When the learner discovers some principle or structure in the make up of the fact under study, the learning becomes sharply focused.

g) Learning depends upon the individual's capacity to learn

Every mathematics teacher is confronted with the necessity of providing for the differences among the pupils. These differences may be due to difference in age, health, intelligence, ability, interests and attitudes. In arranging learning experiences both the logical sequence and the child's capacity to learn should be kept in view.

h) The learning experience should be meaningful

Effective learning is possible when the problem is both intelligelity and of significant interest to the learner. Meaning is developed if the learner sees the worth-whileness of the problem, grasps relationships, and shows its application in many situations.

Motivating the learner

Children learn only when they are interested and put in an effort. Consequently one of the greatest concerns of mathematics teachers is the problem of creating and maintaining interest and effort on the part of their students. The basic condition for effective learning requires pupil's active response and full involvement in the experience offered. Numerous psychological experiments have shown that no amount of pressure, drill, penalization, extra assignments or keeping in after school produces any favourable effect on child's learning. Factors that ensure pupil's motivation are found in the pupil himself, in the teacher and his teaching methods, in the subject-matter and in the society. A few of the suggestions relating to these factors are given here.

A. Factors related to pupils.

1. Pupils possess many basic inclinations and desires which can be utilized by the teacher. A few of these are: curiosity, play, physical activity, mental activity, manipulation, adventure, recognition, social approval, imitation.
2. Pupils are easily attracted to new things - new topics, new ways of working, new problems, etc.
3. The goal should be clear, significant and within the reach of each pupil.
4. Pupils have keen desire to succeed and excel. They like to do things that they could not do before.

5. Economy in effort and thinking is an exciting experience for pupils.
6. Pupils are more interested in things or situation which are true to life.
7. Pupils desire to cooperate, work together and share thinking with others.

B. Factors related to teacher and teaching methods.

Much of the motivation depends upon the teacher and how he controls the forces that act upon pupils. It is the teacher who utilizes the potentialities of each child and the possibilities of the environment for the purpose of teaching:

1. Pupils are influenced by the teacher's own personality and abilities.
    - i) His qualities such as patience, humour, imagination, friendliness.
    - ii) His knowledge of the subject and the pupils.
    - iii) His interest or enthusiasm for his subject.
    - iv) His ability to secure healthy human relationship.
    - v) His ability to relate the subject to life.
2. The teacher can maintain pupil interest in the subject:
    - i) By relating it to its social and practical uses in problems of current living.
    - ii) By relating it to other subjects & vocations.
    - iii) By adapting the materials to the needs and abilities of the pupils
    - iv) By making clear and definite assignments.
    - v) By satisfying his intellectual curiosity.
3. The teacher can stimulate pupil interest by using teaching methods and techniques which encourage wider pupil participation.

4. The teacher can stimulate interest through extra-curricular activities and teaching aids.

C. Factors related to subject-matter.

1. One of the important aspects of mathematics is problem-solving. It stimulates both thinking and action. When the pupil faces a problem he is in a state of uneasiness and tension and on being able to solve the problem he gets satisfaction. As the student gains mastery in problem-solving he finds a strong incentive to continue the study of mathematics.
2. The uses of mathematics is social life and in everyday activities serve as excellent motives for its study. History of mathematics has strong appeal to pupils. In the teaching of certain topics a teacher can find out interesting materials from the history of mathematics. Stories of Money, clock, Weights, Calender have always been of general interest to all. Mathematical games, puzzles, fallacies and optical illusions provide an excellent means of stimulating and fostering interest. The structure and logic of the subject provide most powerful and enduring motivation. The more clearly the pupil understands mathematics the more he wants to learn about it.

D. Factors related to Society

1. The aspirations of the society and its interest in solving its problems scientifically inspire many a young mind to learn mathematics.

2. The various social institutions such as bank, industry, insurance, etc. are the areas which provide for most of the applications of mathematics.
3. Many resources available in a local community can be used as motivating factors in the class.
4. Past achievements of the society inculcate a sense of pride and thus stimulate learning.

Section 4.

TECHNIQUES OF TEACHING MATHEMATICS.

Developing meaning.

Learning is development of meaning and understanding. Pupils learn mathematics most readily through meaningful teaching rather than a machanical process. In a meaningful mathematics programme a balanced recognition is given to the two mutually related and interdependent phases of the subject, namely, its social phase and its mathematical phases. It reveals the concepts and rationale of the processes, and the relationship of processes to each other and bring out their social significance through concrete problems. It is believed that in meaningful learning materials is (a) learned with ease, (b) retained longer, and (c) easily applied to new situations.

The principles underlying meaningful learning are:

1. Learning is directed towards a goal.
2. Learning is organized so that a child can himself discover facts, meanings and generalization through inductive methods.
3. Meaning lies in the individual; each child constructs his own meaning out of his experience.
4. The degree of clarity in meaning depends on child's maturity, interest and motivation, difficulty of the concept and the language in which the concept is presented.
5. Content is presented in proper sequence to facilitate identification of similar elements or pattern or relationships.
6. A wide variety of continuously related learning experiences are provided to extend meanings and to assure needed practice.

7. All learning is pointed towards understanding and not towards memorization.

It is the process of obtaining meaning that is more important. Children must explore and discover rather than be told what to do and how to do. It is through discovery that the child moves from an awareness of a real need for a particular skill, through a variety of concrete experiences, to a recognition of a generalization which he has discovered. Such generalizations, discovered progressively from active and concrete experiences by the child are continuously refined and reorganized into larger generalizations.

Mathematical meaning develops some what as follows:

1. Reference is made to a real situation which matters to the pupil. It involves the readiness of the pupil to accept his own need and to seek new ideas.
2. The pupil "tries" various ways to deal with his ideas. It is a period of experimentation and discovery where in the pupil uses manipulative and pictorial materials to discover the meaning of a particular concept.
3. Symbolic or abstract forms are presented. As new relationships and meanings are discovered, they are expressed in symbols.
4. Generalization are formulated. After symbolization and verbalization of ideas the pupils systematically relate them one to another.

Developing problem-solving ability.

Recent approaches to the teaching of mathematics have been found to lay considerable stress upon critical thinking as an end product of learning. It has been realized that little critical thinking is generated when a student is given a 'model' solution to a problem and is asked to work out a set of similar exercises in mechanical fashion. Rather than giving students all the laws of mathematics and teaching their application to problems, the emphasis is now put upon their discovering laws and supplying reasons for steps. Critical thinking has generally been held synonymous with problem-solving. As a result ability to solve problems have become more important goal in mathematics than the memorization of certain facts used merely as tools in solving problems. Problem-solving is the heart of mathematics, and appears to be the most effective method for learning mathematical facts , concepts and relation.

What is a problem?

Every teacher of mathematics asks the students to solve problems but it does not mean that he tries to develop skills in problem-solving. 'Problem' in the narrow sense is used to indicate a question that is proposed to find an answer. However, not every question proposed is a problem. In the broader meaning a 'problem' is a question which is accepted by an individual as one that he must try to answer and for which the answer or the method of obtaining it is not immediately apparent. If the individual does not experience a definite sense of deficiency and a conscious self-involvement there is no problem for him. A true problem situation must involve novel elements and some search for the "what" and "how".

Problems of mathematics.

The best way for boys and girls to use and improve their abilities to think and reason is through solving many problems in mathematics. Consequently, learning to think with mathematical ideas has important implications for method and content. The method of problem-solving is, itself, content. It is the responsibility of mathematics teacher to provide situations which will produce problems. Problems to be solved may arise in the home or in a social situation. They may be found in text-books or may be formulated by the children or the teachers. They may be real or puzzle type. What is desired is that the students should understand and accept the problem and feel a challenge within them to find a solution.

A mathematics teacher may use problem-solving situations for three purposes: first for the development of mathematical concepts or ideas; second for applying mathematical ideas learnt earlier in new situations; and third; for getting insight into problem-solving method. In all these cases the basic methods involved are the same and the result is a mathematical generalization or the answer to a quantitative question which involves the use of known concepts.

Methods of solving-problems

The basic methods of problem-solving which may be applied at all levels of mathematics instruction can be classified into two groups. First those which involve trial-error and experimental induction and second analytical methods. When a problem is solved by trial and error or experimentation there is uncertainty regarding the answer due to possible error of measurement and the limited number of cases being involved. Analytical methods require higher level of thinking and reasoning and drawing logic inferences.

Characteristics of a good problem situation

All situations are not problem situations. Also what appears to be a problem situation for one person may not be a problem for another. A major task of the teacher then becomes to have a wide range of problems and to relate them to the 'purposes' and 'interests' of individual pupil. At times he may even like to create a problem. Afew of the characteristics of a good problem situation are:

1. There is something new in the situation
2. The goal is clear and definite.
3. It offers a challenge.
4. It is significant.
5. It is within comprehension.
6. It is written in clear, precise and correct words.
7. It is real to life situations.
8. It is related to the 'interest' or of the pupil.

Steps in problem-solving.

1. Becoming aware of the problem

The pupil feels the need of those problems which are within his area of experience. Textbook problems in mathematics often lack in creating interest among the pupils as they are not related to the interest and experience of the pupils. Pupil's becoming aware of a problem also depends on whether he understands the vocabulary and possesses requisite skill in computation.

2. Understanding or clarifying the problem

This involves forming a clear picture of the goal towards which the problem is directed and getting hold of known facts and given conditions. In a mathematics class this involves such questions as:

What is given?

What other relevant facts he knows?

3. Analysis of the problem

This involves finding out facts related to the problem and organizing them into suitable patterns. Emphasis is explicitly put on exploration and organization of facts. Instruction and practice are needed to develop abilities such as, gathering information, checking the reliability of such information, reading to get meanings, visualizing the situation mentally and organizing the information to see what relationships exist.

4. Selecting appropriate method of solving the problem

This involves deciding the procedures that will lead to quick and precise solution. Decisions with regard to use of conjectures, formulae and inductive procedure, etc. are necessary at this stage. The pupils are required to use the accepted rules of mathematics and logic.

5. Solution of the problem

The procedures applied in (4) above result in the solution of the problem. If it does not result in solution the hypothesis is reformulated or new methods for solving are applied.

6. Verification of the result

The process of critical thinking requires that the solution which does not solve the problem be reexamined and if necessary discarded.

Some general directions usually given by teachers in solving problem in algebra are:

a) Know what the problem is
  i) Read the problem
  ii) Know the meaning of the wordsused
  iii) Find what is given and what is to be found.

b) Relate it to a familiar experience.

The problem has a concrete situation (buying) described in words. Visualize or imagine that you are in the problem. Most of such problems lack actual concreteness and fail to interest the pupils.

c) Represent the unknown by symbols.

Organize the data given, by translating it into literal expressions.

d) Write an equation based on the relationship in the problem.

Find expressions and quantities that are equal.

e) Solve the equation.

f) Check the answer.

Failure in problem-solving.

The main causes are:

i) The situation is unfamiliar.
ii) Problem is not read correctly.
iii) Mistakes in fundamental commutation.
iv) Using the wrong computation process.
v) Lack of knowledge of mathematical language and symbolism.
vi) Failure to remember facts, rules, formulas.
vii) Failure to see relationships and apply rules and formulae.
viii) Failure to discard irrelevant facts.
ix) Lack of interest.
x) Failure to check the answer.

Developing proficiency in mathematical skills.

Place of drill in mathematics.

'Practice makes perfect' is an old saying. Repeated practice or drill was a common method of learning mathematics a few decades ago. The teacher used to assign the lesson and the pupils were asked to study and then recite the lesson. Drilling was done to the extent that the skills were automatised. More recently this type of teaching has been more or less discredited. This dislike for drill or automatisation has been, in some cases, carried to other end and a few teachers think drilling as meaningless and futile. The present consensus seems to be that, while practice is a necessary condition for learning, it is not a sufficient one. The emphasis is one motivated drill and meaningfulness. The development of proficiency-accuracy and speed is necessary but even more important is the repeated use of mathematical skills and processes in varied situations.

Importance of drill

1. However well the acquisition and assimilation of knowledge and skills may be, the permanance in learning is only possible by their frequent repetition and application.
2. There are certain operations in mathematics which should be performed not only correctly but also with ease and speed. Systematic practice helps in achieving speed and facility in work.
3. Mathematics is sequential in nature. The ability to acquire new knowledge is conditioned by the mastery the pupil has acquired of the knowledge and skills learnt earlier. Drill strengthens the links between various concepts and skills.
4. It ensures the application of mathematical skills and knowledge, with understanding, whenever situations arise.
5. In order to attend to new problems in mathematics effectively by concentrating on its new aspects, automatization of basic skills useful in solving problems is quite necessary.

Present position

Though to some teachers 'Drill' may be futile, the majority of them seem to be fully aware of its importance in teaching. They use it as a means to fix a skill, pattern or type of the question which has some importance in acquiring new learnings or passing an examination. Moreover, very few teachers differentiate the various aspects of drill, i.e. memorization, review and maintenance. The following are a few of typical situations where drill is used.

1. To help in learning new things.

   The teacher selects a typical sum or problem and solves it on the blackboard with the help of pupils. Then the pupils do more sums or problems which are identical to the one solved by the teacher.

2. To prepare for examinations.

   It is a sort of review of typical examples which are important for the examinations.

3. To revise the work already done.

   At suitable intervals teachers revise the portion that is likely to help in learning new things.

4. As a means to punish.

   If a teacher wants to punish a pupil he often asks him to do more sums. This often leads to distaste for the subject.

A critical examination of the nature of drill and the way it is used reveals the following:

1. It is mechanical and used as a means to memorize formulae, processes and relationships.
2. The purpose is neither clear to teachers nor to pupils. In most cases, examination is the only purpose. When the examination is over, much of knowledge and skill is lost rapidly.
3. It is neither spaced nor supervised.
4. In most of the cases, the pupils follow the pattern shown by the teacher and it is difficult to be sure whether they have really understood the problem.
5. As the teachers in most cases, do not bother to know whether the pupils have done the work (given for drilling) independently or with the help of somebody or have merely copied down from the books of others or from the guides. Validity of such drilling is much doubted.

6. When drilling is used as a means of punishment or as a means of maintaining order it results in creating hatred toward the subject.

Most of the above criticism is directed towards the mechanical, unpurposive and poorly planned drill procedures in teaching. The various aspects of drill - mechanical, review and application are equally important in learning mathematics. It is the teacher who has to decide when and how far these aspects occur in his daily work. Moreover, for each of these the teacher should induce proper motivation in the mind of the pupil and have novelty in each recurring experience planned for pupils.

Objectives of drill

1. To reinforce understanding of concepts, processes and relationships, etc.
2. To gain mastery over the skills once acquired.
3. To gain ability to apply these skills correctly and with ease and speed in new situations.

These objectives aim at increasing the efficiency in solving problems.

Principles of drill

1. Understanding should precede drill.
   Automatization of a skill does not imply understanding and is not likely to last long.
2. Drill to be most effective must be well motivated,
   a) The goal or purpose should be clear.
   b) There should be something new in it.
3. It should identify and eliminate the faulty procedures and errors.
4. It should be adopted to individual needs, level of achievement and rate of work.

5. It should be properly spaced. Drill periods should be short and frequent.
6. The amount of drill should be determined in view of the significance and the difficulty element of the particular skill.
7. It should be properly supervised.
8. The correctness of the response or procedure should be the primary consideration than time and speed.
9. A variety of procedures should be applied for the purpose of creating interest in drill.

Section 5.

ADAPTATION OF INSTRUCTION TO INDIVIDUAL DIFFERENCES

In mathematics, as in any other subject, pupils differ in every phase of learning. They differ greatly in the rates at which they learn, in mental ability, in achievement, in attitude towards subject, and in habits such as hard work, concentration in study, attention. A majority of classes are mixed or unselected groups in which we have students from varying home environments, physical or mental health, varying ages and experiences. Further more, pupils, in a class are expected to begin the course together and complete it as the same time and each pupil studies the same unit and passes on to the next phase of work with all others in the class. To differentiate in a general way these variations in capacity to learn, terms "bright", "average", and "dull" are commonly used. Instruction adapted to the "average" of the class cannot square with the facts of variations among pupils. Good teaching must consider the variations and provide for them in some manner.

Basic principles underlying adaptation of instruction to meet individual differences.

A few basic principles which if followed, will help the teacher provide for individual differences, are as follows:

1. Trial and error and experimentation should by all means be encouraged because it allows pupil to learn his own way. It should however be made sure that it culminates in better skills.
2. Encourage pupils to try and keep on trying. Don't insist on their following a single method or approach.

3. While pupils work in a group permit differences in purpose, interest and achievement to express themselves in such a way that the learning of the individual contributes to the learning of the group.
4. Each pupil much be made to feel secure and have his own share of success.
5. Since the skills can never be mastered all at any one time, once and for all, skills that are needed must be taught or retaught as and when they are needed.
6. Adjust assignments to the needs of the individuals
7. Individual instruction does not rule out social influences, group thinking, and the interplay of mind on mind that can be so very valuable and stimulating for individual learning.
8. The progress of each pupil should be continuously checked.

Below Average Ability Group

In most cases pupils who are able to solve a fewer number of sums on a test are included in this category. A traditional test encourages only memorization of materials and does not take care of all the objectives and whole personality of the child. Therefore, in order to judge pupils status a teacher should use a broader criteria. Some of the things that may help him are

Analysis of pupils past achievements

A readiness test (teacher made)

A diagnostic test (teacher made)

An intelligent test (standardized)

Pupils performance in related fields

Understanding of pupil's needs-

A few suggestions useful in meeting needs of the below average group are given here.

1. Instruction should proceed more slowly and over a longer period of time.
2. Provide many and varied concrete illustration before making pupils generalize.
3. Frequent review or reteaching of concepts in situations familiar to the pupils is necessary to maintain a reasonable level of skill and understanding.
4. Keep problem situations concrete and in natural experimental settings of pupils.
5. Give short and clear assignments to improve quality and mastery.
6. Allow much oral work on the part of the pupils.
7. Give careful attention to vocabulary.
8. Locate specific weaknesses and difficulties and provide more individual help (utilize help of more able pupils when advisable).
9. Arrange for remedial instruction within or outside the classroom.
10. Arrange for pupils to work in small groups.
11. Vary procedures when it is evident that certain procedures are not especially effective.

Above average ability group

The greatest challenge to teachers of mathematics lies in the recognition of and in making provision for those who are above-average in the class. These pupils generally score high on tests, recognize patterns readily, enjoy thinking in abstractions, ask many questions and are impatient with mechanical or rote drill.

While organizing the instruction for these pupils the following points should be kept in view.

1. The instruction should aim at growth in depth.
2. Provide problem situations requiring greater insight, ingenuity and ability to collect and organise data.
3. Avoid unnecessary review and practice.
4. Provide more opportunity for discovery of significant patterns and generalizations.
5. Encourage pupils to read widely.
6. Allow pupils to create thought problems from own experiences.
7. Encourage preparation of models, charts and other exhibits.
8. Correlate concepts with other subject areas.
9. Arrange discussions, competitions and special projects.
10. Allow these pupils to help pupils of lower ability.

Section 6.

EVALUATION OF INSTRUCTION IN MATHEMATICS.

One of the main tasks of every mathematics teacher is to determine the progress his pupils have made towards the achievement of the objectives. In order to do this task effectively it is necessary that he should carefully plan the evaluation programme so as to discover needs, strengths, and growth of the his pupils. A brief pattern for planning the evaluation programme is suggested here. The evaluation programme should follow the following steps:

1. Determination of those significant aims and objective which are to be the goals of instruction.
2. Provision of pertinent behaviour situation to guarantee a valid measure of student reactions.
3. Selection, construction and administration of measuring instrument.
4. Observation of each pupils' behaviour as related to the objective.
5. Interpretation of the results in terms of student needs and as an aid to more effective instruction.

It is expected that the teacher will have in mind the objectives and their specifications in terms of behavioural changes as presented in Section 1. Various techniques are necessary if the progress towards these objectives is to be evaluated. A few evaluation procedures for determining the progress towards each of the objectives are suggested here.

Objective 1: To acquire knowledge of basic concepts, terms, processes and relationships in Mathematics.

Knowledge of basic concepts, terms, processes and relationships in Mathematics can easily be tested through direct questions on the relevant topics. By setting questions of Right/Wrong Type, Multiple choice type, Completion type and Matching type, we can see whether the pupil recognises, recalls, illustrates, defines geometrical terms, detects errors in statements, discriminates between closely related concepts, compares and contrasts terms etc.

Objective II: To develop skill in Mathematical computation.

The fact that the behaviours such as the pupil detects errors in processing, uses mathematical tables and ready reckoness etc. have been developed in the child, can be ascertained through direct questions. Some behaviours like he does computation with reasonable speed, uses short cuts, uses slide rule etc. can be observed in day-to-day class-work and also these behaviours can be judged through questions. The attitudes of precision, checking, self-reliance can be observed in the classroom or in the laboratory when the child is doing practicals or outside the class when the child makes the purchases or transactions in every-day life.

Objective III: To develop skill in using Mathematical instruments and in drawing figures and graphs.

The behaviours that the student measures correctly, draws figures neatly, correctly and with a reasonable speed, etc. can be ascertained by setting direct questions on geometrical constructions. Other behaviour namely selection of appropriate instruments detection of errors in instruments, estimating with accuracy the geometrical measures etc. can be judged on the basis of observations in the classroom. Preparation of charts and graphs can be a good assignment to judge the sense of symmetry and proportion in the students.

Objective IV. To develop an ability to translate mathematical data given in one form into another form.

There can be direct questions on reading a graph, interpreting graphical data in simple language, identifying the scale used, representing a situation in the right kind of graph, translating symbolic terms into simple language, preparing tables or graphs from a direct narrative data, translating given statement into geometrical figure and vice verse, abbreviating a given quantitative statement by algebraic symbols etc.

Objective V: To develop an ability to apply mathematical knowledge and skills in solving problems as they arise in life.

The behaviours namely that the pupil analyses the data, find the adequacy or otherwise of data, selects the relevant data, selects the appropriate method can be tested through direct question. Behaviours namely constructs the problem in a familiar context, estimates the result, verifies the solution, detects errors in solution, solves a problem with other possible alternative methods etc. can effectively be judged in the classroom teaching and other situations.

Objective VI: To develop logical thinking and reasoning.

Direct questions can be set to know whether the pupil distinguishes assumptions from inferences, builds up a logical sequence of the given statements, draws correct inferences from a given hypothesis etc. While teaching the teacher can observe whether the pupil is logical in solving problems, detects the logical fallacies. The behaviour of the child outside the class will tell whether the child recognises the limitations of logical inferences in practical situations.

Objective VII: To develop an ability to locate valid, reliable and relevant information in Mathematics.

Objective VIII: To develop interest in (a) reading mathematical material, (b) mathematical compuations and (c) other mathematical activities.

Objective IX: To develop the following qualities: Accuracy and precision, an attitude of checking computations, neatness in mathematical work systematic presentation of arguments and computations, power of observation, abstraction and generalisation, doing mental calculation with speed.

Objective X: To develop appreciation of the language and history of mathematics and its contribution to life and to the development of other subjects.

These are some of the objectives which can't be measured directly. These mainly relate to attitudes or appreciation as much can be measured indirectly. The assessment in respect of these objectives can be a based on the observations inside the class or outside the class. Though observations are subjective to a high degree and difficult to interpret but for the present they are the only tool. Observations can be supplemented with questionnaires and interest inventories. Through questionnaires and interest inventories we can know whether the child takes pleasure in the study of articles on mathematics, reads mathematics books other than the prescribed ones, knows the various mathematical sources, takes pleasure in mathematical games, takes active part in the Mathematics Club, is interested in preparing charts, graphs, devotes more time to mathematics than the other school subjects, appreciates the use of mathematics in other subjects, in newspapers and also appreciates the contribution of mathematics to life. Through careful observation it can be ascertained whether the qualities like accuracy and precision, systematic presentation, power of observation, abstraction and generalisation, and the attitude of checking, neatness in work, doing mental calculations have been developed.

It is necessary that the evaluation should be treated as an integral part of teaching and characterised by continuity. Testing at the beginning of the year is essential to find out the background in mathematics of the child and his readiness for learning. This may also predict his achievement in mathematics. Testing at the conclusion of the unit or year is necessary to know the achievement made during the time. Sometimes retests and follow up test are given to know what has been retained by the students. The tests commonly used for these purposes can be classified as follows:

Prognastic or Aptitude Tests: Designed to predict a pupils' likely success in a given field.

Inventory or Survey or Pretests: designed to determined a pupils' readiness for new work by measuring his background of previous training and experience

Practice Tests: self-administered for the pupil to check his own progress and needs.

Diagnostic Tests: designed to discover the specific process, skill, type, or level of problem at which pupil mastery breaks down.

Achievement Tests: designed to measure the extent to which the pupil has attained the specific objectives of a course or unit of instruction.

Application Tests: containing unique situation in which the pupil must apply his knowledge or method of thinking and analysing.

These types of tests are both teacher made tests as well as commercial tests (published tests). Both commercial tests and teacher made tests are in use in schools. Both types of tests are good in their own way. They are really to supplement each other. The teacher made tests are to usedvery frequently whereas the published tests should be used on rare occasions. Published tests have the following advantages:

1. These tests are made by experts. They are reliable, valid and have a high discriminatory power.
2. The scoring is objective which saves time and gives accurate scores.
3. Norms are available. The individual performance or group performance can be compared with national or state groups.
4. Retesting is possible using a different form so that a measure of growth is obtained.

The teacher made tests are to be used for the following advantages:

1. Teacher made tests are constructed entirely in the context of local teaching situation.
2. The tests can be constructed to keep pace with curriculum changes.
3. The tests can be constructed with emphasis on understanding and application and appreciations. Most published tests emphasise facts, skills and problem-solving.
4. These tests are inexpensive as compared to published tests.
5. These are inexhaustible in availability and subject to repeated evaluations.

For frequent testing the teachers will have to bank upon the tests constructed by them. In order to make a really effective tool of evaluation, the teachers must be conversant with the characteristics of a good test, types of items generally used and the main steps in the construction of a test.

The following are the salient characteristics of a good test:

1. A test should be as highly objective as possible.
2. A test should be reliable. It should be consistant with what it measures and which it does not measure.
3. A test should be valid. It measures for a selected group of pupils the specific purpose for which it was designed.
4. A test should be economical of the teachers' time as per the construction, administration and interpretation of the test.
5. A test should be 'student-conscious'. The elements of the test should be checked in non-ambiguous language, the reasonable tasks should be set for reasonable periods of time. In test items designed to measure understanding of a principle or ability to apply a principle, computation should be minimised.
6. A test should motivate the best efforts of the students. The questions should be so worded and presented that they will discourage guessing and bluffing. The use of the 'Catch question' should be minimised. However occasional use of such questions might be justified from the point of view of stimulation of accurate thinking. A test

should never be used as a means of punishment but should always tend to create in the mind of the student the attitude that it is worthwhile taking.

7. A test designed to discriminate between students' abilities must provide for measurement of the entire range of abilities. There must be questions easy enough that all students can answer them and question so difficult that perfect scores would be highly improbable; if not impossible.

Types of Questions

The following are the main types of questions

A. Completion.

Fill in the blanks in the following:

1. If the sum of the measures of two angles is 90, then each angle is........... of the other
2. In $\triangle PQR$, if $\angle R$ is a right angle, then PQ is called the .... and RQ and RP are called the ....
3. If an angle bisector of a triangle is also a median, then the triangle is............
4. If $\triangle ABE \overset{or}{=} \triangle DCF$; complete the following statement by supplying the missing symbols. The correspondence A ______ $\longleftrightarrow$ ______ CF is a congruence

$\angle A \overset{or}{=} \angle D$ $\overline{AB} \overset{or}{=}$

$\angle B \overset{or}{=}$ $\overline{AE} \overset{or}{=}$

$\angle E \overset{or}{=}$ $BE \overset{or}{=}$

True/False Type:

Are the following statements True or False?

1. The bisector of an angle lies entirely in the interior of the angle.

1. The bisector of an angle forms two acute angles with the sides of the angle.

3. The diagonals of a rhombus are congruent.

4. When there is one to one correspondence between the vertices of two triangles, then there is congruence between the two triangles.

Multiple choice

Tick mark (✓) the correct answer.

1. Similar triangles are congruent 'What can you say about the statement and its converse

   A. The statement is sometimes were the converse is always true.

   B. The statement and its converse are always true.

   C. The statement is always true, the converse is sometimes true.

   D. The statement is always false, but the converse is always true.

   E. The statement is always false but the converse is sometimes true.

   (He discriminates between closely related concepts).

2. Which of the following sets of st lines can form a triangle?

   A. 5 cm, 3 cm, 2 cm.

   B. 6cm, 3 cm, 2 cm.

   C. 4 cm, 3 cm, 2 cm.

   D. 3.5 cm, 3.5 cm, 7 cm.

   E. All of the above

   (He selects the relevant data)

Analogies

Supply the missing term:

Rectangle: circle = Perimeter: ?

Matching

Write against A, B, C, D the number of statement corresponding to the given terms.

A.	Interest	1. It is the name given to the money borrowed by a Govt. or a trading company.
B.	Banker's Discount	2. It is the difference btween simple interest and true discount.
C.	True Discount	3. It is the sum which is paid for the use of others' money.
D.	Banker's Gain	4. It is the reduction made from the amount of the bill in lieu of its immediate cash payment.
		5. It is the same as the simple interest on the amount of the bill for the number of the days the bill has yet to return.

Essay Type

The ability to organise and integrate information can only be tested through Essay type questions. Ability to solve problems, solving geometrical originals, proving theorems and constructing geometric figures.

Before the teacher actually embarks upon the construction of the test, it must be clear to him that what is to be tested and which is the most effective way of doing it. In the construction of a test, the following steps are suggested:

1. List the objectives which you wish to evaluate.
2. Analyse and state each objective in terms of expected knowledge, skills, applications, reactions and habits.

3. Construct one or more test items for each objective. State each item clearly avoiding trick or catch questions. Select items that have the most significence in measuring objectives. Enough items should be used to make the test comprehensive. Statements should not be lifted directly from the textbook. Each item should be independent of the others. Devise items that require the pupils to apply things learnt rather than merely recalling or recognising facts.
4. Assemble the items for the test according to types of items, relatedness and difficulty. Number the responses consecutively from the beginning to the end of the test.
5. Write clear and concise directions for each type of question.
6. Construct the key. Make sure that no term can be answered simply by referring to another items in the test.
7. Have other instructors evaluate the test.
8. After the test has been administered, analyse the results and revise the items if this seems necessary.

Section 7.

CORRELATION WITH OTHER CURRICULAR AREAS

Mathematics teachers are always faced with the challenge of working out ways and means to bring greater unity into the learning experiences of the pupils. There are many situations in the total school programme which require mathematical facts and skills. Teachers who are alert to these situation may break the barriers between subjects. Correlation is the building up of connections between one subject and another.

While the connection between mathematics and basic sciences such as physics, chemistry, astronomy, etc. is more manifest than the relationships with other fields of study in the secondary school, there are opportunities in almost every category. In fact the situations which would not involve aspects of quantity (processes, measures, forms, position, or problems-solving) are very few.

Some areas of correlation of mathematical concepts (presented in this syllabus) and other subject areas are given here:

Social Studies

A. Desirable mathematical skills for pupils in Social Studies would include:

a) Skill in methods of handling information

i) In analysis - breaking down large themes of masses of data into manageable units and penetrating to irreducible elements.

ii) In synthesis - combining elements, drawing inferences and conclusions, and comparing with previous conclusions and inferences - logical and systematic organization.

iii) In map and chart making and graphic presentation.

b) Skill in memorising results of study-with consciousness of application to new situation by exact reference and analogy.

c) Skill in scientific method - inquiring spirit, patience, exactness, weighing evidence, tentative and precise consulsions.

B. A few mathematical concepts most frequently used in Social Studies are:

Graphic methods and formulas; Equations; Progressiions; Permutations, combinations and probability; Location and scale drawing; Surveying; Interest; Profit and Loss; Discount and other business concepts; Computing devices; Rounding off members; Measurement; Area of plane figures and volume of solids.

C. A few typical situations in which mathematics in useful are as follows:

Comparing density of population; Interpreting picture-graphs in newspapers; Comparing time; Planning journeys by consulting time tables; Calculating fare and time spent in journey; Finding area of different territories; Surveying land and drawing maps and models; Running social and business enterprises; Investment; Income and expenditure problems.

Geography

A few mathematical concepts used in geography are

1. Scale
2. Circle and sphere.
3. Semi-circle
4. Altitude
5. Shape and size.

6. Area
7. Distance
8. Perimeter
9. Horizontal and vertical
10. Statistical data
11. Ellipse
12. Parallels
13. Angle and degree
14. Measurement
15. Graph
16. Tabulation
17. Speed
18. Meridians and parrallels on a sphere.
19. Points and lines on a sphere
20. Spherical distance between two points
21. Plane figures and area
22. Solids their surface area and volume
23. Formulas- Maintaining skills in solving problems through formulas filled to the needs of the individual.
24. Graphs
    idea of a relation- its domain, range and graph.
25. Statistical averages.
26. Units of measure
27. Points, lines and planes
28. Angles and angular measurement.
29. Solids, their surface area and volume.

A few situation in which mathematical concepts are used are:

Scale of a map (e.g. 1" = 100 mile)

The globe

Longitudes and latitudes
Map-drawing.

Shapes and sizes of different countries,continents etc.,

Areas of countries,regions, etc. given in square miles.

The distance of a place from other place.

Length of the boundry line of a country, a continent, etc. Solid surface of the earth

Idea is given by longitudes and latitudes.

Temperatures and rainfalls of different places.

The path on which the earth moves round the sun is elliptical.

Paraallels of latitude

Idea is contained in longtitudes and latitudes

Measurement by chain (measuring fields etc.)

Average rainfall, average temperature tc. of a place.

Tabulation of weather record.

Economics

A few mathematical concepts used in Economics are: Mathematical language and methods - collection of data, -interpretation and discovering laws; percent and percentage; Formulas and equations; Variable; Estimation; Approximation; Statistical concepts; Progression; Graphs; Prabability Area; Measurement; Rate of increase; Relation and function; Maximum and minimum.

A few situations, in which mathematical concepts are -used are:

Changing price levels inflation and deflation; Determinants of employments; Income and prices; Investments; Consumption;Exchange and distribution; Price index; Budget.

Science

Science is an allied subject of mathematics and provides maximum opportunity for correlation. Only a few important concepts of mathematics are, therefore, listed here:-

Mathematical symbolism;Equations;Graph; Formula;Inequality,Indices and Logarithms; Ratio proportion;Elligation;Percentage; Measurement;Area;Volume;Relations and functions; Progression; Point; Line; Plane; Plane figure; Solid;Space; Parallelism; Perpendicularity.

A few situations in which mathematical concepts are used are:

Wave motion and sound;Desnity;Gravitation; Simple harmonic motion; Heat Production; Electric currents; Tension; Pressure; Composition of fuels; Elements; Temperature solubility; Liquids and gazes - composition, pressure and their measurements.

Home Science

A few mathematical concepts used, are: Ration proportion; Percentage; Area; Volume; Approximation; Limit; Solid; Formula; Circle; Measurement; Estimation; Graph.

A few situations in which mathematical concepts are used are:

Comparison of the composition of inspired and expired air and ventilations; Counting milk, picture or bench money; Counting milk bottles and crackers for room; calculation of calories; Preparation of charts for balanced and unbalanced diet; Budget making in homes; Rents and Taxes: Needle work; Embriodery.

Metal Work

A few mathematical concepts used in Metal work are:

1. Shape & size
2. Circle with parts
3. Angle
4. Dimension
5. Scale
6. Measurement
7. Square
8. Perimeter
9. Hexagon
10. Surface and area
11. Transformation
12. Set
13. Parallelism
14. Perpendicularity
15. Fractions and decimals
16. Ratio
17. Volume
18. Tangency and Speed
19. Intersection
20. Limit
21. Axis
22. Cone
23. Sphere
24. Cylinder

A few situation in which mathematical concepts are used are: Shapes and sizes of tools such as pipe-fitting tools, tools for hand forging etc. and other metal things such as bolts, nuts etc.

Drilling holes to lay out

Drawing and sketching

Angle between the two planks of angle-plate.

Dimension of a hole, pipe etc.

Top of a square nut

Top of a haxagon nut

Surface plate

Transformation of measurements from one colliper to another

Set of instruments, tools etc.

Parallel circuit - parallel clamps

The two adj. sides of the square of the square nut.

Reading a micrometer

Efficiency of a machine (out put/in put) cutting mentals in a given ratio.

Volume of a nut, hole etc.

Belts and Pulleys (A belt revolves on pulley)

Hand tools and cutting tools (while cutting metals;

Sheet metal work

Dimension limits in inspecting

Axis of drill

Shape of buffing spindle

Bullets of a gun

Tubes, pipe etc.

Toy-making in school and home

A few mathematical concepts used in Toy making are:

1. Surface - Area
2. Triangle
3. Circle
4. Parallellopiped
5. Perpendicularity
6. Parallelism

7. Cylinder
8. Ellipse
9. Angle
10. Tangency and speed
11. Volume
12. Axis
13. Shape and size
14. Proportion
15. Pyramid
16. Dimension
17. Sphere
18. Similarity
19. Cone
20. Right and Acute angle

A few situation in which mathematical concepts are used are:

Surface of the card-board, top of table etc.

Faces of the top of the lantern in a light house

Bottom of a candle stick, mouth of a cannon, a penny

A magic match-box

Two adj. arms of a cardboard, two adj. arms of a magic match-box.

Opposite arms of a magic match-box.

The middle portion of a barrel organ.

Upper portion of a boat.

Two hands of a clock, spokes of a cycle wheel.

Pulleys and belts.

Volume of a piece of strip wood.
Axis of the wheels of a five-engine.
Shape and size of toys like cars, railway engine, horse, bird etc.
Parts of the bodies of toys like horse, camel, bird etc. are proportionate.
Body of the wind mill, the top of the lantern in a light house.
Dimensions of plateforms, wells etc.

Bullets of a gun.
Similar pivots of the ship cannon, similarity in small and big sized toys such as camels, elephants, birds.
A tent of tournament.
Railway signal: when it is up and when it is down.

Another aspect of correlation of mathematics that a teacher should explore is the applications of mathematics to problems the individual faces in every day life. The applications of mathematics to problems of everyday life are limitless. Two areas have been taken here for illustrating how a teacher can utilize a wide variety of opportunities.

A. MATHEMATICS OF INDIVIDUAL

	Concepts	Skills
1. Caring for health:	i. Average	1. Selecting proper food and other diets
	ii. Ratio	
	iii. Proportion iv. Frequency v. Area	ii. Safeguarding food from harmful things like dust, flies, bacterias etc.
	vi. Volume	iii. Deciding how much area should be used for construction purposes out of the total area for sanitary and hygenic conditions.
		iv. Keeping sanitation.
2. Planning for food clothes and housing	i. Income	i. Estimating the expenditures
	ii. Expenditure	
	iii. Saving	ii. Selecting right type of food, clothes and house
	iv. Cash	
	v. Balance vi. Estimation	iii. Comparing income and expenditure
	vii. Discount.	iv. Making chart or budget of income and expenditure.

3. Saving for future

i. Budget	i. Preparing budgets
ii. Income	ii. Computing percentages and interests
iii. Expenditure	iii. Using interest and other tables.

Concepts	Skills
iv. Saving	iv. Calculating premiums
v. Banking	v. Buying and selling stocks and sharos
vi. Interest	vi. Investing money in bonds, securities, etc.
vii. Profit and loss	vii. To be in the knowledge of the ways of opening an account in a bank, getting insured, etc.
viii. Investment	
ix. Stock and Share	
x. Insurance	
xi. Bcnd dnd saving certificate	
xii. Partnership	
xiii. Discount.	

4. Earning a living

i. Salary or income	i. Determining the amount of wages earned on different bases
ii. Wages	ii. Calculating averages
iii. Profit & Loss	iii. Computing percentages and interests
iv. Commission	
v. Rate of time	

B. MATHEMATICS OF HOME

1. Family income

	i. Salary	i. Determing the amount of money earned on different bases
v. Interest	ii. Income	ii. Comparing the different sources of income and finding the better ones
vi. Average monthly rate.	iii. Prince iv. Commission	

Concepts	Skills
2. Planning for food, clothes and housing i. Record ii. Budget iii. Cost iv. Saving v. Balance	i. Deciding what percent of the income should be spent on various necessities. ii. Deciding how many varieties with what quantities of food should be used. iii. Deciding suitable kind of house. iv. Deciding average quality of clothes. v. Keeping accurate records of earned money and money to be spent.
3. Spending on leisure and recreation i. Record ii. Budget iii. Cost iv. Saving v. Balance	i. Comparing various means of recreation and leisure in respect of their importance for a family. ii. Deciding what percent of income should be spent on leisure and recreation. iii. Deciding how many means of leisure and recreation should a particular family have iv. Keeping accurate records of earning and spending money.

4. Family savings and security	i. Banking ii. Interest iii. Bill iv. Receipt v. Payment vi. Balance vii. Withdrawal viii. Investment ix. Bond x. Insurance xi. Profit xii. Bonus xiii. Certificate of saving. xiv. Postal saving xv. Share and Stock xvi. Partnership	i. Computing percentages and interest. ii. Investing money in postal savings, savings accounts, savings bonds etc. iii. To learn the meanings of words used in post offices, banking and insurance companies. iv. Interpretting insurance tables v. Using interest, insurance and other tables. vi. Calculating premiums, profits etc. vii. Buying and selling stocks and shares.

PART III.

THE PROGRAMME FOR EACH YEAR

Section 8.
Ist Year.

Section 9.
2nd Year.

Section 10.
3rd Year

Section 11.
4th Year.

A knowledge and understanding of the mathematical concepts which the pupils have learnt in Elementary Classes (I-VII) is a prerequisite to the content presented in the following pages. It concludes a redevelopment and extension of the concepts of arithmetic and geometry taught in earlier classes and attempts to provide for continuity in mathematical ideas. It has also taken into consideration age, maturity level, interests and needs of the pupils.

However, what is presented here should be taken merely as a workable breakdown of the scope of the ideas to be developed in each year under the major strands. Each teacher should make necessary changes as suited to the mathematical ability and background of experiences of his pupils.

Concept of Set

A knowledge of the language of sets is a prerequisite to the learning of the content proposed in this guide. It is, therefore, necessary that the pupils should be well acquainted with the set notation, symbolism and operations with sets before they are embarked on the content proposed in this guide. So long as we do not have a changed programme in our elementary schools we shall continue to get pupils in secondary schools who have no idea of set language. Therefore, for a few days in the begining of their instruction during first year following basic notions of the set and operations with set be given, mainly discussing sets constructed from natural numbers and fractions.

1. Notion of a collection ("set") of objects;

   specifications of a set by listing its members, by stating unique conditions for membership.
2. Notation of a set.
3. Idea of sub-set of a set, complement of a set, finite set, infinite set, null or empty set, equivalent sets.
4. Union and intersection of sets.
5. Venn-Euler diagrams.

Ist. Year

I. Concept of Number — II. Concepts Basic to Operations

Concepts		Content
		Rational Number
Rational Number: Integer; Fraction Number line,	1.	Revise pupils' knowledge of integers and fractions (i.e.. rational numbers of arithmetic).
Replacement set.	2.	Extend the number system from integers to rationals:
		a) Provision of replacement sets for equations like $3x = 4$.
Multiplicative Inverse (or reciprocal)		b) Existence of multiplicative inverse (or reciprocal) of all numbers except zero thereby making the operation of division possible.
		c) Filling in of more gaps on the number line.
	3.	Definition ans symbolism:
		a) Definition and representation, (Each rational number may be represented in the for $\frac{a}{b}$, where a and b are integers and be is not zero).
Decimal		b) Rationals as terminating and non-terminating repeating decimals.
		c) Natural numbers and integers expressed as rationals.
	4.	Rationals on a number line:
		a) Show correspondence between points on a number-line and rationals.
Dense set.		b) Set of rational numbers as a dense set.
		c) Comparison of rationals.

Concepts

Operations:
Closure,
Commutativity,
Multiplicative
Identity,
Additive Identity,
Additive inverse,
Distributive law.

Content

5. Condition for two rational number $\frac{a}{b}$ and $\frac{c}{d}$ to represent the same rational number.

6. Multiplication of rationals:
   a) Closure of set of rationals($\frac{a}{b} . \frac{c}{d}$ is a rational number)
   b) Commutativity($\frac{a}{b} . \frac{c}{d} = \frac{c}{d} . \frac{a}{b}$)
   c) Associativity($\frac{a}{b} (\frac{c}{d} . \frac{e}{f}) = (\frac{a}{b} . \frac{c}{d}) \frac{e}{f}$)
   d) Multiplicative identity($\frac{a}{b} . \frac{1}{1} = \frac{a}{b}$)
   e) Multiplicative inverse($\frac{a}{b} . \frac{b}{a} = \frac{1}{1}$)

7. Division of a rational by another as multiplication of the first by the reciprocal of the second.

8. Addition of rational
   a) Closure of set of rationals($\frac{a}{b} + \frac{c}{d}$ is a rational number).
   b) Commutativity ($\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$)
   c) Associativity($\frac{a}{b} + (\frac{c}{d} + \frac{e}{f}) = (\frac{a}{b} + \frac{c}{d}) + \frac{e}{f}$).
   d) Additive identity($\frac{a}{b} + \frac{0}{c} = \frac{ac + b.0}{bc} = \frac{ac}{bc} = \frac{a}{b} . \frac{c}{d} = \frac{a}{b}$).
   e) Additive inverse($\frac{a}{b} + \frac{-a}{b} = \frac{a + (-a)}{b} = \frac{0}{c}$).

9. Subtraction of a rational from another as addition of the first and the additive inverse of second.

10. Distributive law of multiplication over addition($\frac{a}{b} (\frac{c}{d} + \frac{e}{f}) = \frac{a}{b} . \frac{c}{d} + \frac{a}{b} . \frac{e}{f}$)

Concepts	Content
Positive rational, Negative rational, Order.	11. a) Positive and negative rationals $\frac{a}{b}$ is positive if i) a and be are both positive ii) a and b are both negative; $\frac{a}{b}$ is negative if i) a is positive and b is negative ii) a is negative and b is positive b) Order of rationals.
Quotient	12. a) Estimation of quotients of a rationals b) Estimation of the result of operations on rationals. 13. Reducing rationals to lowest.
	Real Number
Real number	1. Show that there are points on the number line for which there no rational numbers. The equation of the form $x^2 = n$ has no solution unless n is a perfect square. $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, and many more are not rational numbers.
	Algebraic Expressions
Expression, Variable, Constant Polynomial Factor Coefficient	1. Revise the following: a) meaning of algebraic expression, variables and constants, and equivalent expressions. b) types of expressions - monomials, binomials, trinomials and polynomials. c) terms such as factor, coefficient,

Concepts	Content
Exponent, base and Power	terms of an expression, replacement-set, exponent, base and power. d) sum, difference, product and quotient of expressions. e) simplification of expressions and removing parentheses. 2. Introduce factional expressions. 3. Polynomials over integers, rationals and reals. Guide pupils to see that every polynomial, in one variable has three characteristics: variable, coefficients and operations.
Degree of an Expression	4. Degree of an expression quadratic polynomials 5. Arrangement of an expression in ascending and descending order. 6. Polynomials with two or more variables. 7. Addition, multiplication subtraction and division of polynomials.
Product Factorization	8. Special products $(a \pm b)^2$, $a^2 - b^2$ and factorization.

III. Concept of Percent and Percentage

	Percent
Percentage Base Rate Rate of growth	1. Revise the following: a) Meaning of percent - base, rate and percentages. b) Formula percentage = base x rate c) Changing percents to fractional equivalents and vice versa.
Profit and loss Discount	2. Problems of percentage-increase or decrease and rate of growth.

Concepts	Content
	ii) The Distance Postulate - To every pair of different points there corresponds a unique positive number.
	3. An infinite ruler:
Scale	i) Idiaof scale on a line
	ii) Coordinate system - coordinate of a point
	iii) Distance between two points
	iv) Ruler postulate
	4. Setting up a coordinate system on a line
	i) Ruler placement postulate
Betweeness	ii) Betweenness.
	5. Line, segments and rays
Line segment ray	i) The line postulate - for every tw points there is exactly one line that contains both points.
	ii) Definition - segment, end point, length of the segment, ray, opposite rays.
	iii) Point plotting theorem Let AB be a ray and let x be a positive number, then there is exactly one point P of AB such that AP = x.
Bisection Mid point.	iv) Mid point of a segment; bisection of a segment.

Concepts	Content
	6. Collinear points. A set of points is collinear if there is a line which contains all the point of the set.
	7. If two different lines intersect, there intersection contains only one point. (Experimental verfication of the theorem)
	8. Parallel lines: Two lines are parallel if i) they are coplanar ii) they do not interesect. Experimental verification of: Throu a given external point there is only one line parallel to a given line.
	9. i) Construction of a right bisector of a given segment with the help of a ruler and a compass. ii) Construction of parallel lines wit the help of set-squares and also ruler and compass. iii) Dividing a segment in a number of equal parts with the help of a ruler, a compass and set-squares.

Plane and Space

Concepts	Content
Plane	1. Definition of space: The set of all points is called space.
Space	
[illegible]	2. Collinear points and coplaner point
Coplanarity	3. Postulates describing planes and space

Concepts	Content
Separation Intersection Flatness (of planes)	a) Every plane contains at least three non-collinear points. b) Space contains at least four non-coplanar points.
	4. Intersection of two lines in one point.
	5. Flatness of planes: If two points of a line be in a plane, then the line lies in the same plane.
	6. Intersection of a line and a plane: If a line intersects a plane not containing it, then the intersection contains only one point.
	7. The plane postulate: Any three points lie in at least one plane and any three non-collinear points lie in exactly one plane.
	8. Determination of a plane by i) a line and a point not on it. ii) two intersecting lines.
	9. Intersection of two different planes
Convex set	10. convex sets A set A is called convex if for every two points P & Q of the set, the entire segment $\overline{PQ}$ lies in A.
Half plane and edge	11. Half planes and edge: A line in a plane cuts the plane into two sets, each of which is convex and is infinite in extent. These two sets are called half planes and the line separating the two sets is called the edge of each of them.
	12. The plane seperation postulate: Given

Concepts	Content
	a line and plane containing it. The points of the plane that do not lie on the line form two sets such that i) each of the sets is convex, and ii) if P is in one of the sets and Q is in the other then the segment $\overline{PQ}$ intersects the plane.
	Angle and Triangle
Angle Side Vertex	1. Angle, sides, vertex: If two rays have the same end point, but do not lie on the same line, then their union is an angle. The two rays are called its sides, and their common end point is called its vertex.
Triangle (side, vertex)	2. Triangles, vertices, sides If A, B, C are any three non-collinear points, then the union of the segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ is called a triangle and is denoted by △ ABC. The points A, B and C are called vertices and segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ sides.
	3. Interior and exterior of an angle:
Interior	a) Let BAC be an angle in a plane E. A point P is in the interior of BAC if i) P and B are on the same side of the line AC and

Concepts | **Content**

ii) P and C are on the same side of the line AB. The exterior of BAC is the set of all points of E that lie neither on the angle nor in its interior.

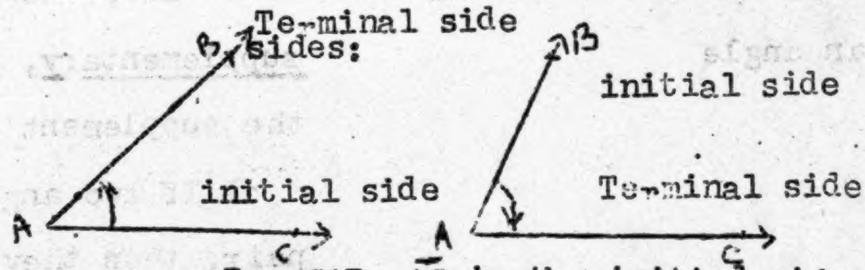
4. Interior and exterior of a triangle: a point lies in the interior of a triangle if it lies in the interior of each of the angles of the triangle. A point lies in the exterior of a triangle if it lies in the plane of the triangle but does not lie on the triangle or in the interior.

Directed angle,

Initial side

Terminal side

5. Directed angles, initial and terminal sides:



In ∠CAB, $\overline{AC}$ is the initial side and $\overline{AB}$ is the terminal side. In ∠BAC $\overline{AB}$ is the initial and $\overline{AC}$ is terminal.

Measure of an angle.

6. Measuring angles with protractor.

7. a) The angle measurement postulate: To every ∠BAC there corresponds a real number between 0 and 180.

b) The number given by the Angle Measurement Postulate is called the measure of ∠BAC, and is written m∠BAC.

Concepts	Content
	8. The angle construction postulate: Let $\overrightarrow{AB}$ be a ray on the edge of the half plane H. For every number r between o and 180 there is exactly one ray $\overrightarrow{AP}$, with P in H, such that m∠PAB = r .
	9. The angle addition postulate: If D is in the interior of ∠BAC, then m∠BAC = m∠BAD + m∠DAC.
	10. a) If $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are opposite rays, and $\overrightarrow{AC}$ is any other ray, then ∠BAC and ∠CAD form a linear pair.
Supplementary angles; Supplement of an angle	b) Supplementary angles: If the sum of the measures of two angles is 180, then the angles are supplementary, and each is called the supplement of the other.
	c) If two angles form a linear pair, then they are supplementary.
	d) Finding measures of the supplements of angles whose measures are given.
	11. Right angles and perpendicularity:
Right angle Perpendicular	a) If the angles in a linear pair have the same measure then each of them is called a right angle. The measure of a right angle is 90.
Complementary angles, Complement of an angle	b) If two rays form a right angle, then they are called perpendicular.
	c) If the sum of the measures of two angles is 90, then the angles are called complementary, and each

Concepts	Content
	of them is called a complement of the other.
	d) Finding measures of complements of angles whose measures are given.
	12. Congruent angles:
Congruent angles.	a) Two angles with the same measure are called congruent angles.
	b) Every angle is congruent to itself.
	13. Theorems:
	a) If two angles are complementary, then both are acute.
	b) Any two right angles are congruent
	c) If two angles are both congruent and supplementary, then each is a right angle.
	d) Supplements of congruent angles are congruent.
	e) Complements of congruent angles are congruent.
	14. Vertical angles:
Vertical angles	a) Two angles are vertical angles if their sides form two pairs of opposite rays.
	b) Vertical opposite angles are congruent
	c) If two intersecting lines form one right angle, then they form four right angles.

Concepts Contents

15. Writing up simple profs.

16. Constructions:

a) Drawing an angle congruent to a given angle with ruler and compass.

b) Drawing parallel lines by making congruent angles.

c) Drawing perpendicular to a line from an exterior point.

d) Drawing perpendicular to a line from an interior point.

e) Bisecting a given angle.

f) Constructing angles of 60°, 120°, 90°, 30°, 45°, 75° and 135° with ruler and compass only.

g) Dividing a segment internally and externally in a given ratio.

h) Constructions of triangles with the following data:

- i) Three sides
- ii) Two sides and the included angle
- iii) Two sides and an angle
- iv) Two angles and the included side
- v) Two angles and a side
- vi) Base, one base angle and altitude form vertex to base.

Concepts	Content
	i) Constructions of right triangles with the following data i) One leg and bypotenuse ii) Two legs
	Polygon and Polygonal Region
Quadrilateral	1. Quadrilateral: If A, B, C, and D are four points of the same plane and no three of these points are collinear, and the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ intersect only at their end points, then the union of these four segments is called a quadrilateral. The four segments are called its sides, and the end points A, B, C, and D are called its vertices. The angles $\angle$DAB, $\angle$ABC, $\angle$BCD, and $\angle$CDA are called its angles. The quadrilateral itself is donated by □ ABCD

Concepts	Content
	The angles of ABC may be denoted for short by A, B, C, and D. Property of convexity: A plane quadrilateral is convex if no two of its vertices lie on opposite sides of a line containing a side of the of the quadrilateral. Consecutive, opposite sides and angles, and diagonal of a quadrilateral.
	Two sides of a quadrilateral are opposite if they donot intersect.
	Two of its angles are opposite if they do not have a side of the quadrilateral in common.
	Two sides are consecutive if they have a common end point.
	Two angles are consecutive if they have a side of the quadrilateral in common.
Diagonal	A diagonal of a quadrilateral is a segment joining two non-consecutive vertices.
	2. Types of quadrilateral:
Trapezium	i) A trapezium is a quadrilateral which has two parallel sides.
arallelogram ectangle	ii) A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.
Square Rhombus	iii) A rhombus is a parallelogram all of whose sides are congruent.
	iv) A rectange is a parallelogram all of whose angles are right angles.
	v) A square is a rectangle all of whose sides are congruent.

Concepts	Content

3. Verification of following properties.
   - a) Each diagonal seperates a parallelogram into two congruent triangles.
   - b) In a parallelogram, any two opposite sides are congruent.
   - c) In a parallelogram, any two opposite angles are congruent.
   - d) In a parallelogram any two consecutive angles are supplementary.
   - e) The diagonals of a parallelogram bisect each other.
   - f) Given a quadrilateral in which both pairs of opposite sides are congruent. Then the quadrilateral is a parallelogram.
   - g) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
   - h) The segments between the mid-points of two sides of a triangle is parallel to the third side and half long.
   - i) If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.
   - j) In a rhombus, the diagonals are perpendicular to one another.
   - k) If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilaterial is a rhombus.

4. i) Construction of a quadrilateral in the following cases:

- a) Given three sides and two included angles.
- b) Given three angles and two included sides
- c) Three sides and one diagonal.

ii) Construction of a parallelogram in the following cases:

- a) Given two consecutive sides and the included angle
- b) Given one side, one angle and one diagonal

iii) Constructions of a rectangle when two consecutive sides are given.

iv) Construction of a trapezium in the following cases:

- a) Given the two parallel sides and the perpendicular distance btween them.

Polygon
Regular
Polygon

5. Recognition and meaning of a polygon (having more than four sides and a regular polygon).

6. Constructions of polygons with given sides and angles.

Circle

Circle

Centre
Radius
Diameter
Circumference
Pie ( )

1. The meaning and recognition of a circle and its parts: centre, radius, diameter and circumference.

2. Drawing circles with given radius or a diameter.

3. The meaning of π (pie) as a ratio of circumference to diameter and being approximately equal to 3.143 (Take $\pi = \frac{22}{7}$)

Concepts	Content
	4. Using the formula for area of a circle (π x radius^2) and formula for the circumference (2x π x radius) in problems from everyday life.
	Solids
	1. The meaning and recognition of the following solids
Prism	i) Right regular prism (including rectangular parallelopiped)
Cylinder	ii) Right circular cylinder
Cone	iii) Right circular cone .
Surface area Volume	2. Using the formulae for surface area and volume of right regular prism, right circular cylinder and right circular cone in problems from everyday life.
	VI. Concepts of Functional Relationship
	Equations and Inequalities
Equation Sentence	a) Revision of formulas for perimeters and circumferences, for areas and for volumes.
Statement	b) Finding the value of an unknown in a formula.
Variable Replacement set	c) Definition of a sentence and a statement, true and false statements.
Open sentence	d) Variables (cf. Algebraic Expressions) and an open sentence - symmetric, non-symmetric and equivalent.
	e) Replacement and replacement set of an open sentence.

Concepts	Content
Graph	a) The meaning of graph
Pictograph	b) Interpretation of pictographs
Bar graph Line graph	c) Making bar and line graphs
Distribution graph	d) Making and interpretting distribution graphs from data of pupil interest.

VII. Concept of Comparison

Ratio and Proportion

Concepts	Content
Ratio	1. Definition of a ratio - terms of a rates.
Equivalent	2. Equivalent ratios.
	3. Expressing a ratio in its simplest form. To find a correct ratio, we must express both the terms of a ratio in the same units of measurement. If a pair of numbers is multiplied or divided by any number except zero, the resulting numbers express a ratio equivalent to that of the original numbers. To find the ratio in simple form that is equivalent to a given ratio, we continue to use appropriate multiplier or divisors until the resulting pair of numbers are the smallest possible whole numbers that have no common factors.
	4. Ratios of equality and Inequality.
	5. Changing ratios to percents.

Concepts	Content
Proportion	6. The meaning of proportion - extremes and means
Extreme	
Mean	7. Properties of proportions

i) In a proportion the product of the extremes is equal to the product of the means:

$$\overbrace{x : \underbrace{y :: 3x}_{x} : 3y}^{x}$$

ii) In a proportion, the terms are in proportion by alternation and also by inversion.

iii) If the numerators of two equal ratios are equal, the denominators are equal and conversely.

iv) If three terms of one proportion are equal, respectively, to the corresponding terms of another proportion, the remaining terms are equal.

v) In a series of equal ratios, the sum of any set of numerators [illegible] to the sum of the corresponding denominators as any numerator is to its denominator.

vi) In a proportion, the terms are in proportion by addition and substraction.

vii) The mean proportional between any two non-zero numbers is the square root of their product.

Concept **Content**

Congruency, similarity and symmetry

Congruence
size
shape

1. Idea and symbol of congruence: Two geometric figures are congruent if they have exactly the same size and shape.
2. One-to-one correspondence and congruence
   a) Matching-up scheme
   b) Meaning of ABC ↔ EFD

Identity-
Congruence

3. Identity congruence: ABC ↔ ABC
4. Deciding upon congruences by inspection
5. Congruent segments and congruent angles:
   a) Segments are congruent if they have the same length
   b) Angles are congruent if they have the same measures.
6. Congruence between triangles:
   a) Given a correspondence ABC ↔ DEF between the vertices of two triangles. If every pair of corresponding sides are congruent and every pair of corresponding angles are congruent, then the correspondence ABC ↔ DEF is called a congruence between the two triangles.

   b) Meaning of $\triangle ABC \cong \triangle DEF$ as six things at once

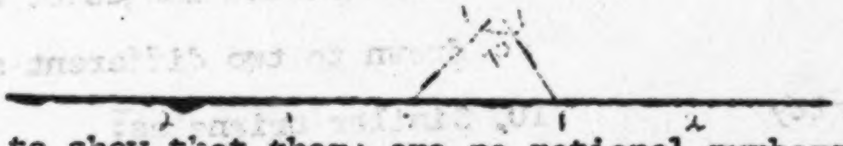
      i) $\overline{AB} \cong \overline{DE}$ or $AB = DE$
      ii) $\overline{AC} \cong \overline{DF}$ or $AC = DF$
      iii) $\overline{BC} \cong \overline{EF}$ or $BC = BF$

Concepts	Content
	iv) $\angle A \cong \angle D$ or $m\angle A = m\angle D$ v) $\angle B \cong \angle E$ or $m\angle B = m\angle E$ vi) $\angle C \cong \angle F$ or $m\angle C = m\angle F$
	7. Congruence postulates for triangles a) Every SAS correspondence is a congruence b) Every ASA correspondence is a congruence c) Every SSS correspondence is a congruence
	8. Application of congruence postulates
	9. Notion of similarity through examination of magnified images of objects or objects drawn to two different scales.
Similarity	10. Similar triangles: If corresponding angles are congruent, the two triangles have the same shape but not the same size.
Symmetry (Point Symmetry Line Symmetry)	11. Notion of symmetry through examination of designs in art, fabrics, jewelry, etc. i) Line symmetry - axis of symmetry. ii) Point symmetry - centre of symmetry
	12. Congruence of symmetrical figures.
	13. Symmetry of an angle, triangle and of a circle.

IInd Year

I. Concept of Number II. Concepts Basic to operations

Real Numbers

Concepts	Content
Real number	1. Revise pupils knowledge of rational numbers and their properties. 2. Extend the number system to real numbers.
Replacement-set	a) Provision for replacement set for equations of the type $x^2 = n$. Use diagram such as:  to show that there are no rational numbers for 2, 3, 5........ b) show the completeness of number line.
Dense	3. a) Show that the set of real numbers contains one number for each point of the number.
Rational Irrational	Some of these correspond to rational points and some to irrational points.
Complementary-set	b) Show that sets of rationals and irrationals are complementary sets of real number
Order	4. Order of reals on the number line. 5. Properties of operations in the system of real numbers. Use geometric analogies, using the concept of number line to show the properties that hold in the system of real numbers.
Absolute value of a real number	6. Absolute value of a real number. 7. Show that set of irrationals is not closed under binary operations.

8. Irrationals as repeating decimals.

9. Addition, subtraction, multiplication and division of an irrational with a rational yield an irrational number.

Algebraic Expression

1. Revise the notions of powers, roots and exponents and extend application to real numbers,

Exponents 2. Laws of exponents

If $a \neq 0$ and if m and n are positive integers, then

Power i) $a^m \times a^n = a^{m+n}$

base ii) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$

$= \frac{1}{a^{m-n}}$ if $m < n$

$= 1$ if $m = n$

iii) $(a^m)^n = a^{mn}$

Negative exponent 3. Negative exponents - if x is any non-zero real number, then $x^0 = 1$; if x is any non-zero real number, and a is any non-zero integer, then

$$x^{-a} = \frac{1}{x^a}$$

Show that laws of exponents hold for integers in general.

4. Use of exponents in scientific notation

Radical 5. Fractional exponents - Radicals symbol

a) Define $\sqrt[n]{x}$; the principal nth root of x terms - index, radicand, root and symbol.

b) Rational and irrational radicals

c) Basic principles used to multiply, divide and simplify expressions involving radicals.

Concepts	Content
	6. Simplification of expressions involving radicals - including rationalisation.
Square root	7. Use of tables for finding out square root and cube roots.

III Concept of percent and percentage

Business Problems

1. Applications of percent in problems from the following areas

Concepts	Content
Banking, Foreign exchange Credit bill	a) Banking, foreign exchange and credit bills Discuss type of accounts and rate of interest, bank draft, promissory note, rate of exchange, credit bill, etc.
	b) Discount. Discuss present worth of the bill and true discount; banker's discount and banker's gain, etc.
Stock, Share	c) Stocks and Shares Discuss the idea of a joint stock company, share, share-holder, dividend, brokerage, market value and face value of a stock etc.
Insurance	d) Insurance Discuss the idea of insurance, types of insurance, premium, policy, face value of policy, beneficiary, bonus, etc.
Tax	e) Taxes Discuss the idea of taxes, types of taxes, their coverage and calculation.
Cooperative society	f) Cooperative societies Discuss the idea of cooperative society and compare it with a joint stock company, capital and shares of a cooperative society dividend.

IV. Concepts basic to Geometric form and position. V. Concepts of Measurement

Concepts	Content
	Proof
Statement True and False Statements	1. Discuss 'if and then' type statements about numbers, geometric objects and activities in daily life.
Negation Symbolism	2. Discuss concepts of definitions, axioms; hypothesis, conclusion.
and(conjunction) or (disjunction)	3. Notion of 'proof'; use of a rule of inference.
	Point and Line
	1. Conditions of parallelism: i) They are coplanar ii) They donot intersect
Skew lines	2. Skew lines and parallel lines.
Parallel lines	3. Theorems on parallel lines: i) Two parallel lines lie in exactly one plane ii) Two lines in a plane are parallel if they are both perpendicular to the same line. iii) Let L be a line and let P be a point not on L. Then there is at least one line through P, Parallel to L. iv) If three or more parallel lines intersect congruent segments on one transversal, then they intercept congruent segments on any other transversal.
Transversal	4. Transversal of two coplanar lines; alternate interior angles; corresponding angles.

Concepts	Content
Alternate interior angles	i) If two lines are cut by a transversal, and one pair of alternate interior angles are congruent, then the other pair of alternate interior angles are also congruent.
Corresponding angles	ii) Given two lines cut by a transversal If a pair of corresponding angles are congruent then the lines are parallel.
	5. The parallel postulates: i) If two parallel lines are cut by a transversal then alternate interior angles are equal. ii) If two parallel lines are cut by a transversal, each pair of corresponding angles are equal. iii) If two parallel lines are cut by a transversl, the interior angles on the same side of the transversal are supplementary. iv) In a plane if two lines are each parallel to a third line, then they are parallel to each other.
	6. Inequalities for segments i) $\overline{AB} \angle \overline{CD}$ if AB $\angle$ CD ii) $\overline{AB} > \overline{CD}$ if AB $>$ CD
	7. Finding a third, fourth and mean proportional between a given number of segments.
Slope	8. Slope of a line

Plane and Space

Proof of the following theorems.

i) If a line intersects a plane not containing it then the intersection contains only one point.

ii) Given a line and a point not on the line, there is exactly one plane containing both of them.

iii) Given two intersecting lines, there is exactly one plane containing them.

iv) In a given plane, through a given point of a given line, there is one and only one line perpendicular to the given line (definition of a perpendicular bisector)

v) The perpendicular bisector theorem.

vi) Through a given external point there is at least one line perpendicular to the given line.

Angle and triangle

1. Sum of the measures of the angles of a triangle.

Theorem

For every triangle the sume of the measures of the angles is 180°.

Corollary

i) The acute angles of a right-triangle are complementary.

Concepts **Content**

ii) For any triangle, the measure of an exterior angle is the sum of the measures of the two remote interior angles.

2. Angles formed by transversal of two coplanar lines; alternate, interior and corresponding angles.

Theorems:

i) If two lines are cut by a transversal, and one pair of alternate interior angles are congruent, then the other pair of alternate interior angles are also congruent.

ii) Given two lines cut by a transversal, If a pair of corresponding angles are congruent then the lines are parallel. (Converse of the above theorems)

3. a) Inequalities for angles

One angle is less then another if ites measure is less.

Exterior angle
Interior angle

b) Exterior angles of a triangle; remote interior angle

i) The exterior angle theroem (proof): An exterior angle of a triangle is greater than each of its remote interior angles.

4. Inequalities in a single triangle.

i) If two sides of a triangle are not congruent, then the angles opposite them are not congruent and the larger angle is opposite the longer side

ii) If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle. (Converse of the above theorems)

iii) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

5. The distance between a point and line:

Distance

a) The shortest distance between a point and a line in the length of the perpendicular drawn from the point to the line.

b) Through a given internal point there is at least one line perpendicular to a given line.

c) Through a given external point there is at most one line perpendicular to a given line.

Altitude

6. a) An altitude of a triangle is a line segment from a vertex of the triangle perpendicular to the opposite side or the opposite side extended.

b) If an altitude of a triangle bisects the side of the triangle, then the triangle is isosceles and conversely in an isosceles triangle the altitude from the vertex (intersection of the congruent sides) bisects the opposite side.

c) The altitudes to the congruent sides of an isosceles triangles are congruent and conversely.

d) The altitudes of an equilateral triangle are congruent.

e) The perimeter of a triangle is greater than the sum of the three altitudes.

7. a) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

b) If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle with its right angle opposite the longest side.

Construction of a triangle in each case with following data:

i) Perimeter and the two base angles.

ii) Base, one base angle and the sum of the two remaining sides.

iii) Base, one base angle and the difference of the two remaining sides.

iv) Perimeter, one base angle and the altitude.

v) Two sides and the median bisecting the third side.

vi) Base, the sum of the remaining two sides and the vertex angle.

vii) Base, vertex angle and the difference of the remaining two sides.

viii) Base, altitude and the vertex angle.

Polygon and Polygonal Region

1. The definition of a polygon:

Polygon If A,B,C,D,...., P are n points of the
same plane and no three of these points
Polygonal region are collinear and the segments AB,BC,CD....
...., PA intersect only at their end points,

then the union of these n segments is called a polygon of n sides or an - gon. The segments $\overline{AB}$, $\overline{BC}$,...$\overline{PA}$ are called its sides and the points A, B, ..., P its vertices.

The definition of a triangular and polygonal region:

A triangular region is the union of a triangle and its interior.

A polygonal region is the union of a finite number of triangular regions, in a plane, such that if two of these intersect, their intersection is either a point or a segment.

2. Postulates:

i) To every polygonal region there corresponds a unique positive real number.

ii) If two triangles are congfuent then the triangular regions determined by them have the same area.

iii) Suppose that the region R is the union of two regions R_1 and R_2. Suppose that R_1 and R_2 intersect in almost a finite number of segments and points. Then $AR = AR_1 + AR_2$.

iv) The area of a square is the square of the length of its edge.

3. Theorems on area of quadrilaterals:

i) The area of a rectangle is the product of its base and altitude.

ii) The area of a right triangle is half the product of its legs.

iii) The area of a triangle is half the product of any base and the corresponding altitude.

iv) The area of a trapezium is half the product of its altitude and the sum of its bases.

v) The area of a parallelogram is the product of any base and the corresponding altitude.

4. Theorems on relationship of areas of polygons:

   i) A parallelogram and a rectangle standing on the same base and between the same parallels have the same area.

   ii) The area of a triangle is half the area of a rectangle or parallelogram, if the triangle and rectangle or parallelogram have the same base and stand between the same parallels.

5. Theorems on quadrilaterals:

   i) Each diagonal separates a parallelogram into two congruent triangles.

   ii) In a parallelogram any two opposite sides are congruent.

   iii) In a paralleogram any two opposite angles are congruent.

   iv) In a parallelogram any two consecutive angles are supplementary.

   v) The diagonals of a parallelogram besect each other.

   vi) Given a quadrilateral in which both pairs of opposite sides are congruent, then the quadrilaterial is a parallelogram.

   vii) If the diagonals of a quadrilaterial bisect each other, then the quadrilaterial is a parallelogram.

VIII) The segment between the mid points of two sides of a triangle is parallel to the third side and half long.

ix) If a parallelogram has one right angle then it has four right angles.

x) In a rhombus the diagonals are perpendicular to one another.

xi) If the diagonals of a quadrilateral bisect each other and are perpendicular, then, the quadrilateral is a rhombus.

6. Polygons: Definition: Let $P_1, P_2, \ldots, P_n$ be a sequence of n distinct points in a plane with $n>3$. Suppose that the n segments $\overline{P_1P_2}, \overline{P_2P_3}, \ldots, \overline{P_{n-1}P_n}, \overline{P_nP_1}$ have the following properties:

a) No two of the segments intersect except at their end points.

b) No two segments with a common end point are collinear.

Then the union of n segments is called a polygon.

The points $P_1, P_2, --, P_n$ are called its vertices and the segments $\overline{P_1P_2}$, $\overline{P_2P_3}$,---, $\overline{P_nP_1}$ are called its sides.

Convex polygon: A polygon is convex if no two points of it lie on opposite sides of a line containing a side of the polygon.

Regular polygon: A polygon is regular if

i) it is convex

ii) all of its sides are congruent

iii) all of its angles are congruent

Properties:

i) A circle can be circumscribed about every regular polygon

ii) A circle can be inscribed in every regular polygon.

iii) The centre of inscribed and circumscribed circles are the same.

The distance from the centre of the regular polygon to each of its sides is called apothem.

7. Constructions:

a) Drawing a triangle equal in area to a rectangle and vice versa.

b) Drawing a triangle equal in area to a parallelogram and vice versa.

c) Drawing a triangle or a rectangle or a parallelogram equal in area to a quadrilateral.

d) Drawing a square equal in area to a rectangle or a parallelogram.

e) Drawing a triangle or a rectangle or a prallelogram equal in area to a trapesium.

8. Construction of a polygon similar to a given polygon on a given side.

Circle

Circle

1. Definition of a circle and its allied parts:

i) Circle: Let P be a point in a given plane, and let r be a positive number. The circle with centre P and radius r is the set of all points of the plane whose distance from P is equal to r.

Concepts	Content
Chord	ii) Chord: A chord of a circle is a segment whose end points lie on the circle.
	iii) Secant: A line which intersects the circle in two points is called a secant of the circle.
Diameter	iv) Diameter: A diameter of a circle is a chord containing the centre.
Radius	v) Radius: A radius of a circle is a segment from centre to a point of the circle.
Concentric circles	vi) Concentric circles: Two or more circles with the same centre are called concentric.
Interior of a circle Exterior of a circle	vii) Interior and Exterior of a circle: The interior of a circle is the set of all points on the plane whose distance from the centre is less than theradius. The exterior of a circle is the set of all points of the plane whose distance from the centre is greater than the radius.
Congruent Circles	viii) Congruent circles: Circles with the congruent radii are called congruent circles.
Central angle	ix) Central angle: A central angle of a circle is an angle whose vertex is the centre of the circle.
Major arc Minor arc	x) Major and minor arcs: Let C be a circle with centre P, and Let A and B be points which lie on C but are not end points of a diameter. Then the minor arc AB is the union of A, B and all points of C that are in the interior of the angle APB. The major arc AB is the union of A, B and all points of C that lie in the exterior of angle APB.

Concepts	**Content**
Semi-circle	xi) Semicircle: Let C be a circle and let A and B be end points of a diameter. A semicircle is the union of A,B and the points of C that lie in a given half pla with $\overline{AB}$ as edge. The points of A and B are the end points of the semicircle.
Measure of arc	xii) Degree measure of the Area: a) The degree measure of a minor arc is the measure of the corresponding central angle. b) The degree measure of a semicircle is 180. c) The degree measure of a major arc is equal to 360 - the measure of the corresponding minor arc.
Inscribed angle	xiii) Inscribed angles: An angle is inscribed in an arc if a) The sides of the angle contain the end points of the arc. b) the vertex of the angle is a point, but not an end point, of the arc. An angle intercepts an arc if a) The end points of the arc lie on the angle, b) all other points of the arc are in the interior of the angle and c) each side of the angle contains an end point of the arc.
Congruent arcs	xiv) Congruent arcs: In the same circle, or in the congruent circles two arcs with the same measure are called congruent.

Concepts **Content**

2. The following theorems :

   i) The arc addition theorem: If B is a point of $\overset{\frown}{AC}$, then $m\,\overset{\frown}{ABC} = m\,\overset{\frown}{AB} + m\,\overset{\frown}{BC}$.

   ii) The measure of an inscribed angle is half the measure of its intercepted arc.

   iii) Any angle inscribed in a semicircle is a right angle.

   iv) Every two angles inscribed in the same arc are congruent.

   v) In the same circle or in congruent circles, if two chords are congruent, then so are the corresponding arcs.

   vi) In the same or congruent circles, chords equidistant from the centre are congruent and vice versa.

   vii) If a line intersects the interior of a circle, then it intersects the circle in two and only two points.

3. Constructions:

   i) Given an arc of a circle, to find its centre.

   ii) Given three distinct non-collinear points, to pass a circle through them.

Solids

1. The meaning and recognition of the solid sphere and pyramid.

Lateral area
Pyramid
Sphere

2. Using the formulae for surface and lateral area of right circular cone, right circular cylinder and right regular pyramids and for surface area of a sphere in problems from everyday life situations.

3. Using the formulae for volume of right regular pyramid and sphere in problems of everyday life.

Concepts	Content
	VI. Concept of Functional Relationship.
Solution set	1. Solution and solution set of an open sentence.
	2. Definition of an equation and an inequalit
Simultaneous equations	3. Equations and inequalities (in one variabl and simultaneous in two variables).
Root	4. Roots of an equation and conditions therefore.
	5. Properties of equations and inequalities.
	6. The equation of a line.
	Graphs
Origin Abscissa	1. The meaning of origin, exis (abscissa and ordinate) and coordinates of a point.
Ordinate	2. Plotting the points on a graph paper
Coordinate	3. Drawing line graphs.
	4. Graph of the solution set of an equation
	5. Graph of an inequality.
	6. Solution of everyday problems by graphs.
	VII. Concept of Comparison
	Ratio and Proportion
	1. Proportional segments
Mean proportional	2. Mean proportions
	3. Application of the ratio and proportion to Geometry, scale drawing and other subjects.
	Congruency, Similarity and Symmetry
	1. Application of the congruence:
	i) Bisector of an angle
	ii) Isosceles and equilateral triangles.
	iii) Quadrilaterals, parallelograms, squares, and rectangles.

Concepts	Content
Geometric mean	2. Meaning of similarity - proportion, proportional, geometric mean between two numbers.
	3. Similarity between triangles: Corresponding angles congruent: corresponding sides proportional.
	4. Measuring heights of trees, poles etc.
	5. The basic proportionality theorem and its converse: If a line parallel to one side of a triangle intersects the other two sides in distinct points, then it cuts off segments which are proportional to these sides.
	6 a) The AAA similarity theorem: Given a correspondence between two triangles. If corresponding angles are congruent, then the correspondence is a similarity.
	b) The SAS similarity theorem: Given a correspondence between triangles. If two pairs of corresponding sides are proportional and the included angles are congruent, then the correspondence is a similarity.
	c) The SSS similarity theorem: Given a correspondence between two triangles. If corresponding sides are proportional then the correspondence is a similarity.
	7. Similarities and right triangles:
	a) In any right triangle, the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.

Concepts	**Content**
	b) Given a right triangle and the altitude to the hypotenuse.
Geometric-mean	i) The altitude is the geometric mean of the segments into which it seperates the hypotenuse. ii) Each leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.
	8. Area of similar triangles:
Area	If two triangles are similar, then the ratio of their areas is the square of the ratio of any two corresponding sides.
	9. Constructions on similar triangles: a) Construction of a triangle similar to a given triangle on a given segment as base. b) Construction of a triangle similar to a given triangle with a given segment as altitude.
	Statistics
Statistics	1. The definition of Statistics.
Data Population Sample parameter Variate Raw data Frequency (Cumulative frequency)	2. Basic concepts in Statistics - Population, sample Parameter and Statistics, Variate - continuous and discrete, Raw data, Frequency distributions.
	3. Graphs of frequency distributions: a) Bar graph b) Pie graph
Frequency polygon	c) Frequency polygon

Concepts	Content
Frequency curve	d) Cumulative frequency curve
Ogive	e) Ogive
	4. Measures of Central Tendency:
Arithmetic mean	a) Arithmetic mean - change of origin, change of scale.
	b) Weighted arithmetic mean
Mode	c) Mode
Median	d) Median, quartiles and percentiles
Quartile	i) by formula
Percentile	ii) by cumulative frequency curve
	e) Properties and uses of these measures.

IIIrd Year

Concepts	Content
I. Concept of Number	**II. Concepts Basic to operations**
	Logarithmic Numbers
Logarithm	1. Meaning of a logarithm. 2. Basic principles of logarithm. i) For any positive real numbers x, y and b, where $b \neq 1, \log_b (xy)$. ii) For any positive real numbers b and x, where $b \neq 1$ and for any real number $n. \log_b (x^n) = n. \log_b x$. iii) For any positive real numbers x, y and b, $b \neq 1$ $\log_b \frac{x}{y} = \log_b y$. iv) For any positive real number x, y, a, and b and $a \neq 1$, $b \neq 1$, $\log_a x = \log_b x. \log_a b$.
Characteristic Mantissa	3. Common logarithms in calculations - characteritic, mantissa.
	Permutation and Combination
Permutation	1. The meaning and symbol of permutation.
Order	2. Arranging letters and other articles (2 to 6 in number) in different orders.
Combination	3. Meaning and symbol of a combination. 4. Difference between permutation and combination.
IV. Concepts Basic to Geometric Form and Position	**V. Concept of Measurement**
	Point and Line
Betweenness	Theorems on betweenness and separation a) If M is between A and C on a line L, then M and A are on the same side of any other line containing C. b) If M is between B and C and A is any point on $\overline{BC}$ then M is in the interier of $\angle BAC$.

Concepts	Content
	Plane and Space
Parallel planes and lines	1. Parallelism of two planes, a plane and line:

Theorems:

a) If a plane intersects two parallel planes, then it intersects them in two parallel lines.

b) If a line is perpendicular to one of two parallel planes it is perpondicular to the other.

c) Two planes perpendicular to the same line are parallel.

d) If each of two planes is parallel to a third plane, they are parallel to each other.

e) Two lines perpendicular to the same plane are parallel.

f) A plane perpendicular to the one of two parallel lines is perpendicular to the other.

g) If each of two lines is parallel to a third line, then they are parallel to each other.

h) Parallel planes are every where equidistant.

2. Perpendicularity of a line and a plane: A line and a plane are perpendicular if they intersect and if every line lying in the plane and passing through the point of intersection is perpendicular to the goven line.

Concepts	Content
	3. Theorem: If B and C are equidistant from P and Q, then every point between B and C is equidistant from P and Q.
	4. The basic theorem on perpendiculars. If a line is perpendicular to each of two intersecting lines at their point of intersection, then it is perpendicular to the plane that contains them.
Existence Uniqueness	5. Existence and uniqueness: a) Through a given point of a given line there passes a plane perpendicular to the given line. b) If a line and a plane are perpendicular, then the plane contains every line perpendicular to the given line at its point of intersection with the given plane. c) Through a given point of a given line there is only one plane perpendicular to the given line. d) The perpendicular bisecting plane of a segment is the set of all points, equidistant from the end points of the segment. e) Two lines perpendicular to the same plane are coplanar. f) Through a given point there passes one and only one plane perpendicular to a given line. g) Through a given point there passes one and only one line perpendicular to a given plane.

Concept	Content
	6. a) Distance to a plane from an external point: The distance to a plane from an external point is the length of the perpendicular segment from the point to the plane.
	b) Theorem: The shortest segment to a plane from an external point is the perpendicular segment.
	Angle and Triangle
	1. Theorems on right triangles:
Right angled Triangle (Right T-iangle)	a) The median to the hypotenuse of a right triangle is half as long as the hypotenuse.
Ratio	b) If an acute angle of a right triangle has measure 30^{0}; then the opposite side is half as long as the hypotenuse.
	c) If one leg of a right triangle is half as long as the hypotenuse, then the opposite angle has measure 30^{0}.
Indirect measurement	2. Using angles and triangles for indirect measurement.
Measurement	a) Distinction between direct and indirect measurement.
	b) Use of Pythagorus Theorem in indirect measurement.
Scale	c) Using scale drawings in indirect measurement.
Similarity	d) Using similar triangles and their properties for indirect measurement.
Elevation	e) Tangent and sine ratios of an angle in indirect measurement.
Depression	f) Angles of elevation and depression.

Concepts	Content
	3. Definitions of point of concurrency and concurrent lines.
	4. Theorems on concurrent lines in a triangle
Excentre	a) The perpendicular drawn to the sides of a triangle from their middle points are concurrent. (Excentre).
Incentre	b) The bisectors of the angles of a triangles are concurrent. (Incentre)
Orthocentre	c) The perpendiculars drawn to the sides of a triangle from the vertices are concurrent. (orthocentre).
Centroid	d) The medians of a triangle are concurrent (centroid).
	Polygon and Polygonal Region
	1. Theorems on quadrilaterals:
Cyclic quadrilateral	a) Bisectors of the angles of a quadrilateral form a cyclic quadrilateral.
	b) Quadrilateral formed by joining the mid points of consecutive sides of a quadrilateral is a parallelogram.
	2. Theorems on polygons:
Interior angles	a) The sum of the measures of the interior angles of a polygon of n sides is (n-2) 180 degrees.
	b) Each interior angle of a regular polygon of n sides has measure $(1-\frac{2}{n})$ 180 degrees.
Exterior angles	c) The sum of the measures of the exterior angles of a polygon is two straight angles or 360 degrees.

Concepts	Content
	Circle
Tangent Point of contact	1. Tangent line to a circle - point of contact A tangent to a circle is a line (in the same plane) which intersects the circle in one and only one point. This is called the point of tangency, or point of contact.
Perpendicular Radius	Theorem: A line perpendicular to a radius at its outer and is tangent to the circle. Theorem: Every tangent to acircle is perpendicular to the radius drawn to the point of contact.
Tangent circle	2. Definition of tangent circles - internally tangent circles and externally tangent circles.
Common Tangent	Two circles are tangent if they are tangent to the same line at the same point
Coplaner	If two tangent circles are coplaner, and their centres are on the same side of their common tangent, then they are internally tangent, If two tangent circles are coplaner, and their centres are on opposite sides of their common tangent, then they are externally tangent.
Chord Perpendicular	Theorem: The perpendicular from the centre of a circle to a chord bisects the chord. Theorem: The segment from the centre of a circle to the mid point of a chord is perpendicular to chord.
Bisector	Theorem: In the plane of a circle, the perpendicular bisector of a chord passes through the centre.

Concepts	**Content**
Secant	3. Secant and Tangent segments If $\overline{QA}$ is tangent to a circle at A, then QA is called a tangent segment from Q to the circle.
Tangent segment Congruent angles	Theorem: The two tangent segments to a circle from a point of the exterior are congruent and determine congruent angles with the segments from the exterior point to the centre.
Secant segment	If a segment intersects a circle in two points , and exactly one of these is the end point of the segment, then the segment is called a secant segment to the circle.
Ray Intercept Arc	Theorem: Given an angle with its vertax on a circle, formed by a secant ray and a tangent ray. The measure of the angle is half the measure of the intercepted arc.
	4. The Power Theorem Given a circle C, and a point Q of its exterior. Let L, be asecant line through Q, intersecting C in points R and S; and let L_2 be another secant line through O, intersecting C in points U and T. Then $QR. QS = QU. QT$ Theorem: Given a tangent segment $\overline{QT}$ to a circle, and a secant line through Q, intersecting the circle in points R and S, then $QR.QS = QT^2$ Theorem: Let $\overline{RS}$ and $\overline{TO}$ be the chords of the same circle, intersecting at Q, then $QR.QS = QU = QU.QT$

Concepts	Content
Cyclic-quadrilateral	5. Definition of a cyclic quadrilateral Theorem: The opposite angles of a cyclic quadrilateral are supplementary.
Limit	6. Concept of limit
Circumference Perimeter Regular polygon Ratio	7. Circumference: The circumference of a circle is the limit of the perimeter of the inscribed regular polygons. Theorem: The ratio of the circumference to the diameter is the same for all circles.
	8. The areaof a circle:
Area Union Limit	a) A circular region is the union of a circle and its interior b) The area of a circle is the limit of the areas of the inscribed regular polygons. Theorem: The area of a circle of radius r is πr^2.
Arc. Length Proportion	9. (a) Lengths of arcs: Theorems: (i) If two arcs have equal radii then their lengths are proportional to their measures.
Measure of arc	(ii) If an arc has measure q and radius r, then its length is $\frac{q}{180} \cdot \pi r$.
Sector	(b) Sectors: Let $\widehat{AB}$ be an arc of a circle with centre Q and radius r. The union of all segments $\overline{QP}$, where P is any point of $\widehat{AB}$, is called a sector. [illegible]B is called a sector, $\widehat{AB}$ is called the arc of the sector, and r is called its radius.

Concepts	Content
	Theorems:
Area	i) The area of a sector is half the product of its radius and the length of its arc. ii) If a sector has radius r and its arc has measure q then its area is $\frac{q}{360} \pi r^2$.
Equation Circle	10. (a) Equation of the circle. (b) Theorems: Every circle is a graph of the equation of the form $x^2 \quad y^2 + ax + by + c = 0$.
	11. Constructions. a) To draw a tangent segment to a circle from a given external point. b) On a given segment to describe a segment of a circle which shall contain an angle congruent to a given angle. c) To circumscribe a circle about a given triangle. d) To inscribe a circle in a given triangle. e) To draw an escribed circle of a given triangle. f) In agiven circle;.to inscribe a triangle equiangular to a given triangle. g) About a given circle to circumscribe a a triangle equiangular to a given triangle. h) To draw a regular polygon in and about a given circle. i) To draw a circle in and about a regular polygon.

Concepts	Content
	Solids 1. Definitions:
Prism Lateral edge Lateral face lateral surface Total	1) Prism: Let E_1 and E_2 by two parallel planes, let R be a polygonal region in E_1 let L be a line which intersects E_1 and E_2 but not R. For each point P of R, let $\overline{PP'}$ be the segment which is parallel to L and joins P to a point P' of E_2. The union of all segments $\overline{PP'}$ is called a prism.
Region Base Altitude	The polygonal region R is called the lower base or simply the base of the prism. The part of the prism that lies in E_2 is called the upper base. The distance between E_1 and E_2 is called the altitude.
	If L is perpendicular to E_1 and E_2, then the prism is called a right prism.
	If A is the vertex of the base of a prism, and A' is the corresponding point of the upper base, then $\overline{AA'}$ is the lateral edge of the prism. If $\overline{AB}$ is an edge of the base, and F is the union of all segments $\overline{PP'}$ for which P is in AB, then F is a lateral face of the prism.
	The union of the lateral faces of a prism is called its lateral surface. The union of its lateral faces and its two bases is called the total surface.

Concepts	Content
Parallelopiped	ii) **Parallelopiped:** A parallelopiped is a prism whose base is a parallelogram region. A rectangular parallelopiped is a right rectangular prism.
Cube	iii) **Cube:** A cube is a rectangular parallelopiped all of whose edges are congruent.
Pyramid	iv) **Pyramid:** Given a polygonal region R in a plane E, and a point V not in E. The pyramid with base R and vertex V is the union of all segments $\overline{VQ}$ for which Q belongs to R. The altitude of the pyramid is the (perpendicular) distance from V to E.
Cylinder	v) **Cylinder:** If in (i) R is a circular region the resulting solid is called a cylinder.
Cone	vi) **Cone:** If in (iv) R is a circular region then the resulting solid is called a cone.
Sphere	vii) **Sphere:** Let P be a point and let r be a positive number. The sphere with centre P and radius r is the set of all points of space whose distance from P is equal to r.
Cross-section	viii) **Cross section of solid:** A cross section of a solid is the intersection of the solid with a plane parallel to the plane of the base.

2. Geometry of the sphere:

a) Allied parts of a sphere

Chord: A chord of a sphere is is a segment whose end points lie on the sphere.

Concepts	Content
	Secant: A line which intersects the sphere is called a secant of the sphere. **Dismeter:** A diameter of the sphere is a chord containing the centre.
	Concentric spheres: Two or more spheres with the same centre are called concentric.
Great circle small circle	**Great and small circles:** The intersection of a sphere with a plane through its centre is called a great circle of the sphere. If the plane does not pass through the centre, its intersection with the sphere is called a small circle.
	b) **Theorem:** The intersection of a sphere with a plane through its centre is a circle with the same centre and the same radius.
Longitude latitude	c) Meridians and parallels on a sphere. d) Longitude and latitudd.
	Theorems on solids
	a) All cross sections of a triangular prism are congruent to the base Corollary: The upper and lower bases of a triangular prism are **congruent**.
Congruent Area	b) All cross sections of a prism have the same area. Corollary: The bases of a prism have the same area.
Region	c) The lateral faces of a prism are parallelogram regions. Corollary: The lateral faces of a right prism are rectangular regions.

Concepts		Content
Base Vertex	d)	Every cross section of triangular pyramid, between the base and the vertex, is a triangular region similar to the base. If h is the altitude and k is the distance from the vertex to the cross section, then the area of the cross section is equal to $\frac{k^2}{h^2}$ times the area of the base.
Altitude	e)	In any pyramid the ratio of the area of a cross section to the area of the base is $\frac{k^2}{h^2}$, where h is the altitude of the pyramid and k is the distance from the vertex to the plane of the cross section.
	f)	If two pyramids have the same base area and the same altitude, then the cross sections equidistant from the vertices have the same area.
Volume	g)	If two pyramids have the same altitude and the same base area, and their bases lie in the same plane then they have the same volume.
	h)	The volume of a triangular pyramid is one third the product of its altitude and base area.
	i)	The volume of a pyramid is one third the product of its altitude and base area.
	j)	Every cross section of circular cylinder has the same area as the base.
	k)	Given a cone of altitude h and a cross section made by a plane at a distance k from the vertex. The area of the cross section is equal to $\frac{k^2}{h^2}$ times the area of the base.

Concepts		Content
	l)	The volume of a circular cylinder is the product of its altitude and the area of its base.
	m)	The volume of a circular cone is cone is one-third the product of its altitude and the area of its base.
	n)	The volume of a sphere of radius r is $\frac{4}{3} \pi r^3$.
	o)	The surface area of a sphere of radius r is $4 \pi r^2$.

VI. Concept of Functional Relationship

Equations and Inequalities

Concepts		Content
Quadratic equation	1.	Meaning of a quadratic equation and a quadratic inequality.
Imaginary number	2.	Meaning of an imaginary number - solution of the equation $x^2 = -1$.
	3.	Cmplets quadratic equation standard form of a quadratic equation.
Quadratic inequality	4.	Incomplete and pure quadratic equations.
	5.	Solution of quadratic equations by a) factor method b) completing the square (including the general equation $ax^2 + c = 0$) and c) Formula
	6.	Graphs of quadratic inequalities.

Trigonometric Ratios

Concepts		Content
Ratio	1.	Ratios of the legs and hypotemuse of a right triangle.

Concepts	Content
	2. Relations among the trigonometric ratios:
	a) $\text{Tan } \angle A = \frac{\text{Sin } \angle A}{\text{Cos } \angle A}$
	b) $(\sin \angle A)^2 + (\text{Cos } \angle A)^2 = 1.$
	c) $(\text{Sec } \angle A)^2 = 1 + (\tan \angle A)^2$
	d) $(\text{Cosec } \angle A)^2 = 1 + (\text{Cot } \angle A)^2$
	3. Trigonometric ratios of 0^0, 15^0, 20^0, 45^0, 60^0, 90^0, 120^0, 135^0, 150^0, 180^0, 210^0, 225^0, 240^0, 270^0, 300^0, 315^0, 330^0, 360^0 amg;es.
	Relations and Graphs
Order-pair	1. Ordered pairs and coordinates
Coordinate	2. Relation - Binary relations
Binary relations	a) Relation as negation of a sentence defining a relation
Negation	b) Construction of ordered pairs on the basis of a relation and vice versa.
Relation	c) Domain and range of a relation
Range	d) Graph of a relation
inverse	e) Inverse of a relation and its graph.
	3. Compound sentences and their graphs.
	VII. Concept of Comparison
	Statistics
Range	Measures of dispersion
Mean deviation	1. Variability
Variance	2. Range
Standard-	3. Semi-inter quartile range (quartile deviation)
deviation	4. Mean deviation
	5. Variance and Standard deviation
	6. Peoperties of the above measures and their uses.

4th Year

Concepts	Content
I. Concept of Number II. Concepts Basic to Operations	
	Progressions
	1. Definition of a progression
Arithmetic-Progression	2. Arithmetic progression:
	a) Definition
	b) First term, common difference, nth term
	c) Finding the nth term and sum of n terms
Arithmetic mean	d) Arithmetic mean between two numbers.
	3. Geometric Progression:
Geometric-Progression	a) Definition
	b) First term, common ratio, nth term
Geometric mean	c) Finding the nth term and sum of n terms
	d) Geometric mean between two numbers.
	Permutation and Combination
	1. Permutations
Permutation	a) The fundamental principle: if a first event can happen in h_1 ways and if, after it has taken place, a second evemt cam happen in h_2 ways, then the two events can happen in $h_1 h_2$ ways. In general if, after the first two events have taken place, a third can happen in h_3 ways, a fourth in h_4 ways, and finally an nth in h_n ways: then the n events can happen in the order indicated in $h_1 \ h_2 \ h_3 \ h_4$ --- h_n ways.
	b) Definition of permutation
	c) Permutation of n things taken r at a time.
Factorial	d) Meaning of the factorial and its notation.

Concepts	Content
	2. Combinations
	a) Definition
	b) Combination of n things taken r at a time.
	3. Relation between nP_r and nC_r.
	Binomial Theorem
Mathematical Induction	1. Concept of mathematical induction
	2. Binomial expansion for positive integral index.
	3. Proof of the binomial theorem for positive integral index.

VI. Concept of Functional Relationship

	Functions and Graphs
Correspondence	1. Correspondence - many to-one.
	2. Correspondence between ordered pairs of real numbers and points in a plane.
Function	3. Function and inverse of a function.
Inverse of a function	a) Notation of a function
	b) Independent and dependent variables
	c) Graph of a function
Periodic function	4. Periodic functions
Even function	5. Even and odd functions
Odd function	6. Trigonometric functions
Trigonometric-function	a) Variations in the values of trigonometric functions over a range of 0^0 to 360^0.
	b) Graphs of trigonometric functions.
Exponential-function	7. Exponential functions and graphs
	8. logarithmic functions and their graphs
Logarithmic-Function	9. Exponential equations and their solutions
	i) $a^{x+2} = a^5$
	ii) $a^{x+y+3} = a^2$ $b^{x+2y} = b^6$.

Concepts	Content
	Trigonometry
Trigonometric-ratios	1. Trigonometric Ratios of Compound and multiple angles. 2. Solution of right triangle problems.
	VIII. Concept of Probability **Probability**
Probability	1. Meaning of probability 2. Mutually exclussive, dependent and independent events. 3. Theorems : a) The probability that some one of a set of mutually exclussive events will occur in a single trial is the sum of the proababilities of the separate events. b) The probability that all of a set of independent events will occur is the product of their seperate probabilities.

Allocation of time

While schools should be left free to adjust their schedule of work to suit their conditions and circumstances, it was thought necessary to suggest a broad pattern to which they may generally conform and which they may useof as a guide for fixing the time of actual instruction in school.

Total No. of instructional days in a year	200	
Duration of each period for instruction	40 mts.	
Number of instructional Periods per week for General Mathematics	1st Year	6
	2nd Year	6
	3rd Year	4
	4th Year	3
Total No. of periods for General Mathematics in a year	1st Year	200
	2nd Yeaf	200
	3rd Year	150
	4th Year	100

PART IV

TEACHING UNIT

Percent and Percentage

Sets

Equations and Inequalities

Graphs

Relations, Functions and Graphs

Line and Plane

Angles and Triangles

Congruence

Circle

The objectives of the curriculum can be realised only when appropriate learning experiences are selected. The principles of learning which have been cited in a previous section in this guide precisely indicate the characteristics of desirable learning experience. It may however, be noted here that it is rather impossible to infer the experiences the pupils have when they face a situation. The learning experiences included here are therefore, only suggestive and teachers are expected to make necessary adjustments keeping in view the range of individual differences among pupils, the resources and time available and the purposes accepted.

For the purpose of organising the learning experience it has beenselected a numberof 'Units of subject matter', mainly because they ensure the unity ofthe content-both within a topic and within different topics. In planning the units, the folowing steps have been followed:

1. The specific objectives of the unit were determined keeping in view the objectives ofthe curriculum.
2. The content of each'Unit' was analysed into different categories such as terms, concepts, deffinitions and principles.
3. The content was specified into important understandings,
4. Suitable activities were then suggested keeping in view the class-room situation.

The activities selected contributeto words the attainment of the objectives kept in view for the unit. Most of the activities are pupil activities and lay emphasis on pupil's doing. A few activities are for the teacher and wey help him in directing group work or individual work. The activities that can possibly be carried out outside classroom have not been specified because they can only be indicated in general terms such as preparing a bulletin board, collecting and displaying materials, using library and the specfic content for such activities as to be worked out by the individual teacher.

Since the try out of these units was not taken up at this stage specific evaluation materials in the form of various tests could not be prepared for each unit.

PERCENT AND PERCENTAGE

Objectives

1. To help the pupil to understand the concept of rate of growth as an application of percent.

2. To enable the pupil to use meaningfully the principles of percentage in applied problems in profit and loss, interests, stocks and shares, discount, insurance, taxes and budgeting.

3. To help the pupil to develop computational skill in computing profit or loss in transactions, in computing interests, in finding dividends earned by stocks and shares, in computing the amount of tax on income or property and in reading tables.

4. To enable the pupil to express percents in group or picture form.

5. To encourage the pupil recognise percent problems as they arise /various forms in other subjects which he is currently studying. /in

6. To develop positive attitude towards problem solving in percent.

7. To help the pupil to appreciate the use of percent in our environment, business, industry and Government and personal affairs.

Terms

1. Rate of growth.

Provious Knowledge:

i) Profit & Loss.

1. Cost Price
2. Selling Price
3. Profit
4. Loss
5. Market Price
6. Discount or Commission
7. Overhead Charges
8. Net Price
9. Cross and net prifit.

ii) Simple Interest

1. Loan
2. Principal
3. Rate of Interest
4. Time
5. Interest
6. Amount.

Compound Interest

1. Compound Interest
2. Time -- yearly, half yearly 4 Quarterly etc.
3. Instalment buying.

Discount

1. Bill of exchange
2. Amount - sum due, face value
3. Present worth
4. Drawing date of the bill
5. Due date of a bill
6. Legal due date- grace days
7. True discount
8. Date of discounting or encashment
9. Banker's discount
10. Bankers gain.

Stocks & Shares

1. Stock - debenture - bond
2. Share
3. Dividend
4. Market value of stock
5. Stock at par, at premium or above par, at discount as below par.
6. Brokerage

Insurance

1. Insurance - insurer, insurred
2. Policy - joint policy
3. Face value of a policy
4. Premium
5. Dividend & bonus
6. Paid up policy
7. Lapse
8. Expectation of Life.

Taxes

1. Tax; levy: duty (custom); income tax; tariff, surcharge, sales tax, purchase tax, property tax.
2. ad valorem
3. Slab system

Instalment buying

1. Instalment
2. Carrying charge
3. Down payment

Co-operative Society

1. Co-operative Society
2. Loan

Budget

1. Budget - individual or family
2. Balancing of the budget.

UNDERSTANDINGS

1. The meaning of the terms mentioned under the heading of "terms".

2. Understanding that to find the percent of increase or decrease, the amount of the increase or decrease is found, the result is compared with the original.

COMPOUND INTEREST

3. Understanding that money is said to be lent at compound interest when interest, as it becomes due, is retained by the borrower and the interest for the next period of time is calculated on the increased principal.

4. For the fraction of a time period, the rate is taken proportionately.

5. Understanding of the following principles:

   i) $\text{Amount} = \text{Principal} \left(1 + \frac{\text{Rate}}{100}\right)^{\text{Time}}$

   ii) $\text{Compound Interest} = \text{Amount} - \text{Principal} = \text{Principal} \left[\left(1 + \frac{\text{Rate}}{100}\right)^{\text{Time}} - 1\right]$

   iii) $\text{Principal} = \text{Amount} \left(\frac{100}{100+\text{Rate}}\right)^{\text{Time}}$

6. Understanding that compound interest for any year is greater than that for the year before.

7. Understanding that the amount after a certain year is counted or taken as the principal for the following period of time.

DISCOUNT

8. Understanding that the trade discount is calculated on the marked price while the cash discount is calculated on the selling price.

9. Understanding that, in case of successive discounts, each successive discount is computed on the resultant net price after the former discount, rather than all added together as one discount.

10. Bankers Discount is always greater than True Discount and their difference is called Banker's Gain.

11. Understanding that the calculation of True discount and Banker's discount differ from the cash and trade discounts in the sense that in the formers the time element enters in the calculations.

12. That the legal due date falls three days after the nominal due date. These three days are called days of grace.

13. Understanding that the use of discounting a bill from a banker is that we get immediate use of money which, otherwise, was postponed for some time.

14. Understanding of the following principles:

    i) True discount is the interest on Present worth of the bill.
    ii) Sum due = Present Worth+True Discount.
    iii) Banker's Gain = Banker's Discount-True Discount
    iv) Banker's Gain=Interest on True Discount.

STOCKS & SHARES.

15. Understanding that the capital of a trading company may also be termed as stock.

16. That a shareholder is not entitled to withdraw his money from the Company at any time, although he can sell his share to someone else at the buyer's price like any other commodity.

17. That when a person sells his stock or share to some other person, he has to surrender his right to receive interest to latter.

18. That the market value of a stock does not remain constant but it varies from time to time owing to market conditions though the nominal value or face value of the stock always remains constant.

19. That the Company receiving the loan issues to the lenders an acknowledment guaranteeing therein a certain percentage of interest to be regularly paid at stated intervals.

20. Share holders are part proprietors of a company and get dividend only out of the profits of the company while stock holders are loan creditors getting a fixed rate of interest as agreed upon.

21. That the sale and purchase of stocks or shares is effected through the medium of a broker who makes a certain charge for his services called Brokerage.

22. While buying we add the brokerage to the market price and subtract in case of selling.

INSURANCE

23. Understanding that Insurance is a plan to share the loss of money incurred due to death, accident, fire or any other catastrophe or for securing economic protection for some specified period of time.

24. Understanding that rates of insurance are different in different conditions in different types of insurance.

25. Understanding that premium may be paid monthly quarterly, semiannually and annually.

26. Understanding that an insured is to give a list of beneficiaries in order who may have the face value of the policy after the death of the insured.

27. Understanding that a policy is a contract between the company and the insured.

28. Understanding that life expectancy is the average number of years of life remaining at any age.

29. Understanding that the amount of premium will depend on the expectancy of life at the present age of the insured.

TAXES.

30. Understanding that 'Tax' is a compulsory payment for public benefits.

31. Understanding that property tax includes tax on houses, buildings, land, etc.

32. Understanding that custom duty is an amount which is charged while bringing articles from foreign countries into this country and vice versa.

33. Understanding that "Ad valorem duty" is determined by the value of the article.

34. Understanding that personal property taxes are based on movable property owned by the citizens.

35. Understanding that sales tax is a direct tax which is levied on the sale of any article while purchase tax is a tax which every purchaser is to pay while purchasing an article. In practice sales tax is charged from the purchaser.

36. Understanding that income tax is the tax levied on the income of an individual.

37. Understanding that rates of income tax are fixed on a slab system.

INSTALMENT BUYING

38. Student should understand that the instalment buying is based on following points:

    i) The amount of cash payment
    ii) No. of instalment payment
    iii) Amount of each payment
    iv) Carrying charges which includes interest, cost of keeping records.

39. The student should understand that instalment buying is always expensive.

40. The student should understand that the object purchased -----serves a collateral for the loan and may be repossessed if a defaul* in payment occurs.

BUDGETING

41. Understanding that a budget is a plan for the use of money whether it be the income of an individual, a family or a government.

42. Understanding that the use of the budget in managing the home is a method of relating what one needs or wants to the amount that he has to spend.

43. Understanding that the habit of recording and classifying expenditures is one of the best aids to wise spending and systematic saving.

44. Understanding that wise purchasing for the home makes it necessary to know the quality of the article purchased and to take advantage of buying in quantity.

SKILLS & ABILITIES

1. To carry out computations at a reasonable speed for finding dividends, calculating the total cost when list price and ad volorem duty are known, computing total cost on purchase and net price on the sale of stocks, calculating premium on different types of policies finding duty on articles imported etc.

2. To solve problems involving percentage.

3. To draw and read graphs on percentages.

4. To use compound interest tables, ready reckoners, income tax & other tax tables, rate tables of premium etc. and to read and interpret stock quotations.

5. To locate information about a particular topic from different sources e.g. inquiring rate of discount from banks.

6. To discriminate between closely related concepts, processes and principles e.g. discount, bankers, discount, simple interest, compound interest.

ATTITUDES

1. An attitude of comparing things (rates e*c.) in every day life and business problems like buying and selling, landing money, comparing different sources of dividends etc.,

2. An attitude of wise investment.

3. An attitude of proper payment of taxes - avoiding evasion of taxes.

4. An attitude of having planned expenditure.

5. An attitude of encouraging cooperative movement.

SUGGESTED ACTIVITIES

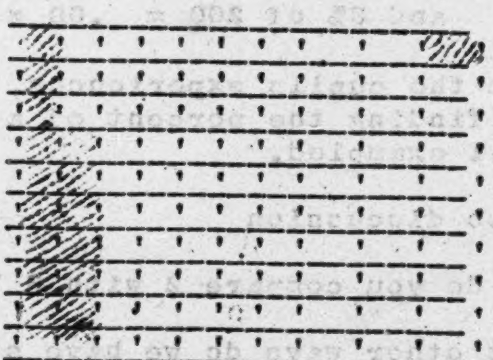
PERCENTAGE.

1. Have each pupil bring news clippings, advertisements, etc. that deal with percents. Discuss the meaning of such statements as "This material is 70% wool and 50% silk", "sales last month were 8% up", and so on.

2. Explain to the students that fractions and decimals have previously been used as ways of comparing numbers or parts of a whole. Explain them by illustrations that this type of comparison can also be made by means of percents.

3. Show that fractions are always compared to 100 when related to percents.

   A graph black board or squared paper can be used to illustrate percents. Mark off a spare for 100 squares.



   Shade some portion of the area and show that this is a part of 100. Show that one shaded square = $\frac{1}{100}$, 17 shaded squars = $\frac{17}{100}$

4. Ask the students to perform the inverse operations, i.e., changing percents to decimals and fractions, and help them to arrive at a rule for the same.

5. Explain to the students that some percents are so frequently used in newspapers and business that it is necessary to learn their fractional equivalents. They should be encouraged to do the same and afterwards a test may be given to the students to check their competency in showing relationships of equivalent forms.

6. An illustration like the following will help the pupils to develop the concepts of finding the percent of a number.

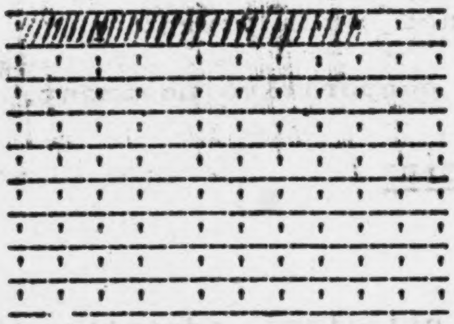


Fig. 1

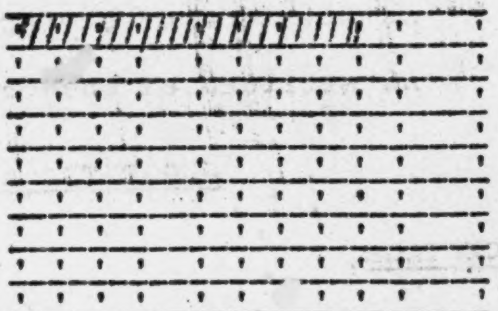


Fig. 2

To find 8% of 200

How many squares are shaded in Fig. 1? In Fig. 2 ?
What is 8% of 100 squares

Since 8% of each figure is 8 squares, 8% of 200 squares = ______________ squares.

Instead of adding the number of shaded blocks in both the figures, we may arrive at the answer by multiplying the number by the percent written as a decimal. Thus

8% = .08

and 8% of 200 = .08 x 200 = 16

From the pupils experiences, continue to develop the concept of finding the percent of a number by illustrating with several examples.

7. Have a class discussion

How do you compare 2 with 8 ?

What other ways do we have of showing comparison by diversion ?

Use a variety of situations in which pupils must decide which comparison is to be made.

8. Develop with the pupils an understanding of the following methods of finding the whole when a part is given:

Example: A boy bought a 30% interest in a boat for ₨.15. What was the value of the boat?

(i) First method

30% of the boat= ₨.15
1% of the boat= ₨.½
100% of the boat= ₨.½ x 100 = ₨.50.

(ii) Second Method: Change 30% to $\frac{3}{10}$

$\frac{3}{10}$ of the boat = ₨.15

$\frac{1}{10}$ of the boat = $\frac{1}{3}$ x ₨.15 = ₨.5

$\frac{10}{10}$ of the boat = 10 x ₨.5 = ₨.50

(iii) Use the formula, P = RB

Here P = ₨.15, R = 30%, B = 7

∴ ₨.15 = 30 x B

or ₨.15 = $\frac{30 \times B}{100}$

or 30B = ₨.1500

or B = ₨. $\frac{1500}{30}$ = ₨.50.

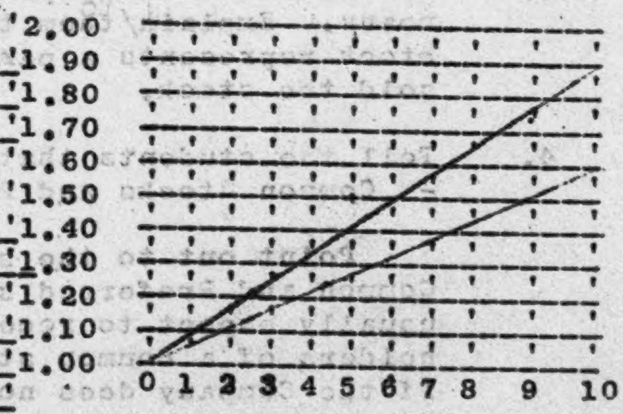
9. There should be given a sufficient practice in each of the three types of percentage problems to the students for complete mastery.

When solving social problems involving percentage, encourage the pupils to solve them with the use of formula and without it. Exercises, based on experiences the pupils may have had, may have more meaning for them. Give them the same type and encourage them.

COMPOUND INTEREST

1. The table given below gives the amount of ₨.1 at Simple Interest and Compound Interest at 6%. Draw the line graph showing their comparative growth.

Year	Amount at Simple Interest	Amount at Compound Interest
0	1.000	1.000
1	1.060	1.060
2	1.120	1.124
3	1.180	1.191
4	1.240	1.262
5	1.300	1.358
6	1.360	1.419
7	1.420	1.504
8	1.480	1.594
9	1.540	1.689
10	1.600	1.791



Stress the students that it is profitable to lend money at Compound Interest than at Simple Interest.

2. Explain the students that if the lender would add the payment for its use to the money lent and invest the new principal at the same rate of interest, the next interest payment would be slightly larger and is called Compound Interest.

3. Explain to the students that the interest may be added to the principal annually, semiannually, quarterly, monthly, etc. The number of times interest is added to the principal in a year is called the "frequency of conversion".

4. Solve some problems on the board by the direct method, i.e., finding principal after every interest period, and then develop the formula for computing compound amount.

$$\text{Compound Amount} = \text{Principal} \left(1 + \frac{\text{Rate}}{100} \right)^{\text{Time periods}}$$

Explain to the students that compound interest can be got by subtracting the principal from the compound amount.

5. Have pupils work out several problems using the interest formula. Tell the students that they can check their results from interest tables.

Demonstrate the use of compound interest tables in the class. Show the pupils how a table that shows amounts compounded annually can be used to find amounts compounded semiannually quarterly.

Give the students sufficient practice in solving the problems on compound interest and encourage them to check the results through interest tables.

STOCKS AND SHARES

1. Discuss with the pupils the necessary steps in the organisation of a Joint Stock Company for the purpose of doing a business.

2. Tell the students about the functions of a Joint Stock Company.

3. Be sure that the pupils understand what a share of stock means. Explain to them that the ownership of a share of stock represents a part ownership in the Company that sold the stock.

4. Tell the students that there are two types of the stocks - Common Stocks and Preferred Stocks.

   Point out to the students the difference between Common and Preferred stocks. Preferred stockholders usually expect to receive a stated dividend whereas the holders of a common stock receive a variable dividend. If the Company does not earn sufficient profits, no one receives dividends.

5. Explain to the students that just as it is necessary for a family to borrow money at the time of need, so is it necessary for a Joint Stock Company to borrow money. The Company issues bonds when they need to borrow money.

6. Obtain a sample bond and explain to the pupils that since a bondholder is a lender of money, the bond is an evidence of a debt and bears certain facts similar to a promissory note.

7. Have pupils bring to the class the financial page of the newspaper and teach them how to read news regarding sale of stocks, bonds, etc. Tell the students that the place where stocks, bonds, etc. are sold is called a Stock Exchange.

8. Tell the students that stocks and bonds are sold through brokers. The class may invite a local broker to talk with them about stocks and how they are bought and sold.

9. Have pupils bring quotations of the stock values of different Companies from the newspapers to the class. Have the pupils to compute the prices of different types of shares of stock in a company and how much more or less the stock is worth each day.

Give them a sufficient practice in solving problems regarding buying and selling of Stocks and Bonds.

INSURANCE

1. Have on display pictures which will depict the risk of losing something. A newspaper clipping telling about a burning house may be brought to the class and read.

   Ask the boys. "Suppose the building had been yours. How would that have affected your family? Do you know of any way in which your family can protect itself against such losses".

2. Develop the definition of insurance based on the concept that it is the mutual sharing of economic risks. Point out that through insurance people buy protection against financial loss for a specified period of time.

3. Have the children make an outline in their notebook showing the two groups - Personal and Property-into which insurance may be divided.

4. Make a list of all types of insurances on the board. Point out that each type of insurance is based on the specific needs of an individual or family.

   The class may obtain different kinds of policies to compare and classify.

5. Explain the meaning of "Life Expectancy". Consult an insurance mathematician to find out if our expectancy is increasing or not.

6. Describe the students the typical life insurance needs.

7. Bring the insurance tables and tell the students how to use them. Sample problems may be used to explain how the rates are computed from tables for varying amounts of insurance.

8. Discuss with the class the factors which influence the costs of fire and automobile insurance and why they do so.

   Compare fire insurance rates for different parts of the city and automobile insurance rates for drivers of different ages.

9. Invite an insurance agent to discuss insurance and the role of mathematics in "Insurance". Let the class clear their doubts about any type of insurance.

TAXES

1. The need for imposing taxes should be developed through class discussion. The pupil should learn that taxes are collected to provide for the common needs of all the people, such as schools for public, building of roads, bridges, dams, protection, health, sanitation, public welfare and relief, and maintenance of the government.

2. Emphasize the fact that government services are received by everyone in return for the tax money.

   Have the pupils make a list of the benefits and services derived because of taxation. Explain to the students that it would be impossible to have many services which are provided by public expense if it were not our system of taxation.

3. Several references on the subject for assigned reading should be put in the class room.

   Ask the pupils to read from the references the services given by each government unit. Ask them also to read the sources of income for each government unit and enlist them.

4. Have pupils arrange a display of pictures of taxable articles, clippings from newspapers, etc. Stress the taxes which are familiar to pupils, and use much class discussion.

5. Following the discussion of the topic of income tax, sets of booklets giving information in income tax may be provided to the students. From this experience the pupils will learn the meaning of gross and net income and what constitutes deductions, such as taxes paid, bad debts and personal exemptions.

Explain to the students that the tax is computed on the taxable income after all deductions are made. From the use of the tax tables, show the pupils that the rates of income tax are graduated and increased for higher incomes.

6. Explain to the students that the local governments depend to a great extent on taxes from estate and personal property. So they should know how the tax rate is determined and computed.

Have pupils solve problems finding the tax to be paid on local property. Solving problems with various rates helps the pupil increase his understanding of relative values.

7. Tell the students that sales tax is a compulOary tax which is levided on specified items purchased and delivered.

Have pupils bring in information on sales tax and items covered under it. Give them a sufficient practice with problems on sales tax on different articles.

8. A class discussion should discuss the meaning and purpose of duties; first, to protect the industries of our country; second, to provide revenue for the country.

Have pupils bring in information on duties and tHe teacher should explain the meanings of the terms etc. used in the same.

INSTALMENT BUYING

1. Ask the students to bring clippings from newspapers and magazines regarding "instalment buying" and explain the meaning of those to the students.

Explain to the students that a consumer who uses this type of purchasing should know the following things:

a) The amount of down payment (cash)

b) Number of instalment payments.

c) Amount of each payment.

d) Carrying charge (interest, cost of keeping records, etc).

Point out to the students that instalment buying is expensive since the persons, who use this type of purchasing are actually borrowing the money to pay for the articles and consequently must pay interest on the money borrowed. In addition, most retailers attach a service charge to cover costs of keeping records, and other costs.

[illegible] the words "Instalment buying should be reserved for the purchase of nece-

3. Explain to the students that the object purchased serves as collateral for the loan and may be repossessed if a default in payment occurs.

4. Give the students problems involving a comparison of costs of cash buying and instalment buying.

   Ask the students to enlist the advantages and disadvantages of instalment buying. Determine whether the advantages outweigh disadvantages on particular articles.

5. Tell the students that the rate of interest on balances owed on instalment buying are usually higher than interest rates on money borrowed from other sources in order to cover the risk of some of the cases which do not pay instalments upto the end.

6. Make a list of the articles which can be bought on instalments.

BUDGETING

1. Plan a monthly or yearly family budget and make a circle graph to show wise distribution of income.

2. Collect sample budget forms from various consumer groups including government.

SETS

Previous Knowledge

1. Concept of whole numbers.
2. Four fundamental operations with the members.
3. Concept of size and shape
4. Concept of units of measurement
5. Preliminary idea of algebraic structures
6. Concept of simple geometric figures and terms.

Objectives

1. To acquire knowledge of concepts, terms, processes and relationships in sets.
2. To develop logical thinking and reasoning.
3. To develop the ability of the students to apply the knowledge of sets in various branches of mathematics, i.e. Arithmetic, Algebra and Geometry for being more acurate and precise.
4. To develop interest in Mathematics in general and sets.

Terms

1. Set: infinite set, null set, subset, finit set, member of the set
2. One-to-one matching
3. Union of sets
4. Intersection of sets
5. Complement of a set
6. Mathematical sentence
7. Symbols such as $\in, \cup, \cap, \phi. \subset, \supset.$

Understandings

1. The meaning of the element of a set and that the elements of the same set have a common property.
2. The definition of a set and element of a set and that how to represent a set symbolically.
3. The meaning of a sub set through various concrete examples i.e. a basket having different fruits, a cupboard having books on different subjects, etc.
4. The meaning and symbol for a null set.
5. Meaning of union and intersection of sets through diagrams and examples like;
   If $A = (X : 0 \underline{\angle} x \angle 5)$ and $B = (y ; z \angle y \angle 7)$;
   them $A \cup B = (z : 0 \angle z \angle 7)$ and
   $A \cap B \; (k : 2 \angle k \angle 5)$

6. Comparison of two sets and one-to-one correspondence.

7. The meaning of a finit and an infinite set.

8. Operations with sets and properties.

Teaching Points

1. The meaning of a set

2. The sign language of sets

3. Comparing sets

4. One-to-one matching

5. Introduction of finit, infinite and empty sets

6. Sets within sets

7. Set diagrams

8. Operations with sets: Unions of sets, intersections of sets

9. Laws of operations with sets

10. Sets and sentences

11. Sets of ordered pairs and operations.

Skills

1. Defining terms

2. Comparing sets

3. Recognising the type or kind of set

4. Making diagrams to represent the relationships of the given pairs of sets

5. Using operations.

Activities

1. Ask the pupils to write the members of each of these sets:

   a) The set of odd numbers less than 10.

   b) The set of months beginning with the letter A.

   c) The set of names of States of India beginning with the letter M.

2. Describe the following sets in words:

   a) K = 5, 10, 15, 20, 25

   b) M = 3, 6, 9, 12, 15, 18

   c) R = Tuesday, Thursday

3. Let the students answer the following questions about members of a set:

a) Is a whale a member of the set of all fish?

b) Is meat a member of the set of foods containing protein?

c) Is 64 a member of the squares of whole numbers?

4. Ask students to compare the following sets to find if they are equal:

a) 5, 10, 15, 20, 25 and 5, 15, 25, 10, 20

b) 1, 3, 5, 7, 9 and the set of odd numbers less than 10.

c) The set of students in your form with blue eyes and the set of boys in your form.

5. Ask, which of these pairs of set have a one-to-one correspondence:

a) a,b,c,d,e and 3,7,4,8,5

b) The set of odd numbers below 20 and the set of prime numbers below 20

c) The set of even natural numbers less than 20 and the set of odd natural numbers less than 20.

6. Let them answer the following:

What is the number of members of each of these sets?

a) The set of states of India

b) The set of crops of U.P.

7. Ask, which of the following are equivalent sets:

a) 1,2,3,4,5,6 and 7,14,21,28,35,42

b) 5, a, 7, m and F, R, P, B, C, M

8. Which of the following are infinite sets?

a) The grains of sand on all the beaches of the world.

b) The population of the world.

c) The natural numbers evenly divisible by 13.

9. Which of the following sets are empty sets?

a) The set of foreign born prime ministers.

b) The set of men 15 feet tall.

c) The set of squares of odd numbers that are even.

10. Ask the pupils to make up one subset for each of the following sets.

a) w, x, y, z

b) The set of holidays in a year.

c) The set of four-sided geometric figures.

11. Copy and complete this table, showing the number of subsets for each set:

Set	Number of members of the original set	Number of subsets
a	1	
a,b,c	3	
a,b,c,d	4	

12. Have students write a universal set for the following sets:

a) The set of students at Delhi University who study Mathematics

b) The set of whole numbers between 10 to 20.

13. Let the students make venn diagrams to represent the relationships of the given pairs of sets:

a) The set of players on your schools foot ball team; the set of players on your school's swimming team.

b) 1, 2, 3, 4, 5 and 2, 3

c) The set N of odd numbers; and set p of prime numbers.

14. List the members of the union of each pair of sets:

a. A = a, b, c, d ; B = b, d, f, h

b. X = 2, 4, 6, 8, 10 ; Y = 3, 6, 9, 12

15. List the members of the intersection of each pair of sets.

a) a, b, c, d, e ; a, e, i, o, u

b) The set of prime numbers less than 10; the set of odd members less than 10.

16. Suppose that the universe is the set of natural numbers less than 10, or U = 1,2,3,4,5,6,7,8,9 and of X = 1, 2, 3, 4, 5 what are the members of x' i.e. the complement of X;

17. Consider the following data about a universe and several sets of numbers:

U = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

X = 2, 4, 6, 8

Y = 1, 3, 5, 7

$Z \cap Y$ = 1, 3

$Z \cup Y$ = 1, 2, 3, 5, 7, 9.

Use venn diagrams to find:

a) the numbers in set Z

b) $n(x) - n(z \cap x)$

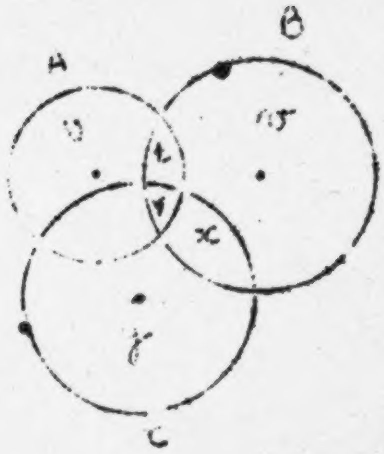
c) $n(z \cup x) - n(z \cup y)$

18. Let the students find.

a) (AUB) UC

b) AU (BUC)

c) (A∩B) ∩C.



EQUATIONS AND INEQUALITIES

Objectives

1. To help the pupil develop the concept of an equation/inequility as a sentence both true and false.
2. To help the pupil to express a verbal statement in symbols like greater than ($>$), equal to ($=$) and less than ($<$) and vice versa.
3. To help the pupil to form equations from verbal statements.
4. To help the pupil learn how to solve an equation/ inequality using four fundamental operations.
5. To help the pupil represent and solve an equation/inequality graphically.
6. To help the pupil develop the habit of checking the solution of an equation/inequality.
7. To help the pupil use equations in solving problems.

Terms

A. Previous knowledge

1. A mathematical sentence.
2. True and false sentences.
3. Skill in reading and writing sentences.
4. Skill in using the symbols 7, = , ∠, and
5. The definition of a "set" - members of a set.
6. Expression.
7. Factors.
8. Order.
9. Exponent - idea
10. Number line - coordinate of a point on a number line.
11. Plotting points, in a coordinate plane.
12. Distributive law (in relation to numbers).
13. Ways of describing a set.

B. New terms

1. Statement - True and false statements.
2. Open sentences
3. Variable
4. Replacement and replacement set.
5. Solution and solution set.
6. Equation - members of an equation (left and right members).
7. Inequality - members of an inequality (left and right members).
8. Roots of an equation.
9. Coefficient.
10. Symmetric and non-symmetric open sentences.
11. Equivalent equations)
) Equivalent open sentences
12. Equalent inequalities)
13. Term
    i) Absolute term (or constant)
    ii) Like terms.
14. Graph of the solution set of an equation.
15. Graph of an inequality.

Understandings:

1. a sentence that is either definitely true or definitely false is called a statement.
2. a variable is a symbol which serves to hold a place in a sentence for the name of some definite things and in mathematics, we often use a letter of the alphabet as a variable.
3. when, in place of a variable, we use the name of one of the things it represents, we say that there has been a "replacement" in place of the variable.

4. the set of things from which the replacement for the variable is chosen is called a 'replacement set'.

5. the numbers which when replaced for the variable make an open sentence a true statement are called the solutions of an open sentence and set of numbers constituting the solutions of the open sentence is called a solution set.

6. an open sentence which says that two things are equal is called an equation.

7. an open sentence which says that two things are unequal is called an inequality.

8. the members of the solution set of an equation are called the roots of the equation.

9. in case of an equation, the expressions on the two sides of equal sign are called "members" of the equation; the expression on the left of the equality sign is called 'left member' and the expression on the right is called "right member" and similarly the members on the left and right of the sign of inequality are called respectively left and right members of inequality.

10. any one of the factors of a product is called the coefficient of the remaining.

11. an open sentence is said to be symmetric if its left and right members can be interchanged and in the contrary case it is a non-symmetric open sentence.

12. if two equations are such that the solution set of one is the same as solution set of the other, the equations are called equivalent equations.

13. if two inequalities are such that the solution set of the one is the same as solution set of the other the inequalities are called equivalent inequalities.

14. sentences which involve some variable (an unknown which can be replaced by a suitable value or values) are called open sentences.

15. a term that is a number, such as 6, is called an absolute term, or a constant.

16. like terms have exactly the same letter for variables, and in like terms a variable appears with the same exponent.

17. the graph of the solution set of an equation in the coordinate plane consits of all the points whose coordinates belong to the set of ordered pairs in the solution set.

18. an open sentence becomes a statement when the symbol in it (unknown) is replaced by the name of some definite thing.

19. possession of roots in an equation depends on the replacement set.

20. if the left member and right member of an equation are operated on by multiplying each by the same non-zero number, an equivalent equation is obtained.

21. if the left member and the right member of an equation are operated on by adding the same number to each or by subtracting the same number from each, an equivalent equation is obtained.

22. all inequalities show the order of two expressions.

23. if you operate on both members of an inequality by multiplying each by the same positive number, an equivalent inequality is obtained.

24. if you operate on both members of an inequality by multiplying each by the same negative number an equivalent inequality is obtained by changing ">"to"<" or by changing "<" to ">".

25. if you operate on left and right member of an inequality by adding or subtracting the same number from each, an equivalent inequality is obtained.

26. to graph the solution set of an equation

a) Find ordered pairs of numbers that may be used as replacements for x and y to make the equation true. These ordered pairs are members of the solution set of the solution.

b) Draw a pair of coordinate axes X'X and Y'Y. Plot the ordered pairs of numbers as points in the coordinate plane. Remember that first number in a pair is the x - coordinate; and second the y - coordinate.

c) Draw a line that passes through all of them. Test some of the points on the graph to see if their coordinates belong to the solution set of the equation.

d) Test whether points not on the graph have coordinates that do not belong to the solution set.

27. a term is any of the following

a) A number

b) A variable

c) The production of two terms, or of a term and an expression.

d) The quotient of two terms, or of a term and an expression.

27. the root of an equation, when replaced for the variable, satisfies the equation.

Skills

1. Recognising true and false sentences and statements.
2. Recognising a variable in an open sentence.
3. Finding replacements from a replacement set such that open sentences will result in true or false sentences.
4. Recognising equations and inequalities.
5. Finding solution sets for equations and inequalities from a given replacement set.
6. Finding roots of an equation.
7. Recognising equivalent equations and inequalities.
8. Solving equations and inequalities.
9. Drawing graph of the solution set of an equation.
10. Drawing graph of the solution set of inequalities.
11. Evaluating expressions (for different values of the variable).
12. Translating verbal language to symbolic form.
13. Checking the computations and the solution set of open sentences.

Suggested Activities

1. Have pupils discuss many types of statements such as:

   A. Mr. Nehru was the first prime minister of India

   B. $4 + 8 = 6 + 6$.

   C. $\frac{1}{2} \times \frac{3}{4} + \frac{4}{8} = \frac{7}{8}$

   D. 8 more than 20 equals the product of 4 and 7.

   Let pupil judget them as true or false.

2. Emphasize that statements of equality may be judged true or false.

3. Introduce the symbol " $\neq$ " is not equal to

4. Discuss statements such as

a) X is talled than Y

b) $17 > 15$

c) $\frac{1}{2} > \frac{1}{4}$

5. Have pupil use symbols " $>$ " and " $<$ " in expressing sentences.

6. Have pupil express the meaning of sentences such as

a) $14 > 10$

b) $1.5 < 4.7$

c) $4\frac{5}{7} > \frac{3}{8}$

7. Have pupil discuss truth or falsity of sentences.

8. Have pupil realize that he cannot judge sentences such as, 'He is the member of my family,' '$4x = 0$' as true or false until he knows to what the pronoun 'He' or 'x' refer.

9. Have the pupil discuss the idea of solution set.

10. Give practice in finding replacements for 'x' from the solution set.

11. Have the pupil learn some terminology - domain, variable, value, solution, root, satisfy, solution set.

12. Have pupil find solution set for the equations and inequalitites when domain is given.

i) $2n = 5$ if domain in each case is

ii) $x > 2$ (2.1, 2.2, 2.3, 2.4)

iii) $n + 1 > 3.3$.

13. Have pupil use principles of Addition, subtraction, multiplication and division for solving equations.

14. Ask pupil to copy the following and add a word or a phrase that will make each sentence a true statement.

a) In an equation the algebraic expression on the left side of the euqal sign is called the ....

The process of finding a number to replace x in the equation $5x = 15$ which will make the equation a true statement is called

....

c) The number which when substituted for x makes the equation $5x = 15$ a true statement is called the .... of the equation.

d) The number which when substituted for x makes the equation $5x=15$ a true statement is said to be..............

15. Ask pupil to write the nunerlas 1-8 on paper. Against each numeral write I if the corresponding sentence is an identity and E if the corresponding sentence is an equation.

1. $10 = 11$ C
2. $-3x = 0$
3. $2x + 5 = 5$
4. $5 + a = a + 5$
5. $a^3 + b^3 = (a+b)\ (a^2+b^2+-ab)$
6. $t = -t$
7. $x^3 - 3ax^2 = 3a^2x + a^3 = 0$
8, $\dfrac{x^2 + \frac{1}{x^2} + 2}{(x + \frac{1}{x})2} = \dfrac{(x - a)^2}{x^2 - 2ax + a^2}$

16. Ask the pupil to use the principles of inequalities and write the solution set and draw the graph of each of the following:

a) $2x - 1 < 3$ d) $2(3x - 2) \geqq 5x + 8$

b) $3x + 2 \ngtr 5$ e) $-2x + 3 \leq 0$

c) $x - 7 \leq 5 - 3x$ f) $3 \leq \frac{3}{5}x - 6$

17. Ask the pupil to give practice in formulating equations for written statements such as:

1. Six more than a number gives ten

$6 \quad + \quad x \quad = \quad 10.$

2. Four times a number decreased by one is twenty-one.

$4 \quad x \quad - \quad 1 \quad = \quad 21$

18. Ask the pupil to select the best answer. On a sheet of paper write each question number and beside it the letter of your choice of answer.

1. What value of x satisfies the equation $x-1 = 3$.

a) 2 b) 3 c) 6.

2. Five times a number decreased by 6 is 49. What is the equation for this ?

a) $5n + 6 = 49$

b) $5n-6 = 49$

c) $5n = 43$

3. The root of $7x -2 = 3$ is:

a) an odd number

b) a fraction

c) an even number.

4. If m and n stand for two numbers what represents 5 less than their product.

a) $5 - m n$ b) $m n - 5$ c) $5 m n - 5$ d) $5 - 5 n n$

19. Ask pupil to select the best method for solving each of the following equations. A means addition, S mean subtraction, M means multiplication and D mean division.

(1). $3 x + 1 = 9$

a) A then D

b) D then S

c) S then D

(2) $5x - 1 = 14$

a) A then D

b) D then S

c) S then D

(3) $\frac{1}{2}x + 8 = 12$

a) S then M

b) A then D

c) A then M

20. Ask pupils to give proof for true or false statements in each of the following:

a) $2x + 4 > 16$, then $x > 6$

b) If $3x - 2 \leq 7$, then $x \leq 3$

c) If $-3y + 4 \geq 5$, then $y \ngeq -1$

21. Make a list of problems in arithmetic and geometry where use of equalitions is possible

22. Represent graphically problems on inequalities.

23. Make a list of topics of other subjects where equations are used.

24. Ask the pupils to

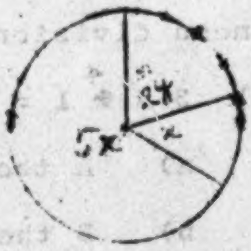
a) study the drawing

b) write the equation or equations necessary for solving the problem, and

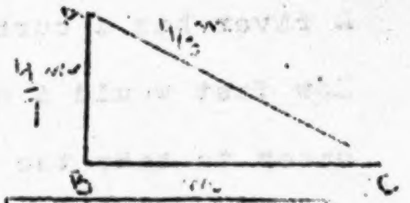
c) solve the equation or equations for the variable or variables.

Some of the equations will be first-degee, some second-degree, and some will be system of two equations with two variables. Remember if there are two variables, two equations are required for finding the solution.

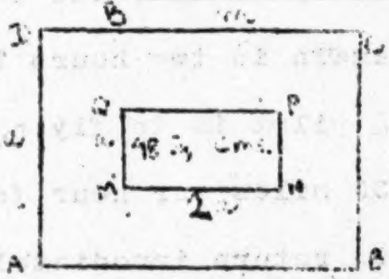
a) if $x+2y = 140$
find x and y.



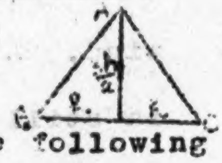
b) Find m, if the perimeter of the triangle ABC is $82\frac{1}{2}$ cms.



c) Find the lengths in cms. of sides AB and AD in rectangle ABCD.



d) Find h if the area of this triangle is 121 square cms.



25. Ask the pupil to solve the problems of the following types involving age, number relationships, geometric figures, time, rate and distance, wind and current, rate of work and mixture. Check your answers by carefully substituting the value or values in given problems.

i) If three times a certain number is increased by six times its reciprocal, the result is eleven. Find the number.

ii) Two-thirds of my age 3 years ago, plus one-six of my age 3 years from now, will equal my age 4 years ago. How old am I ?

iii) The first angle of a trinangle is $\frac{4}{5}$ of the second angle, and the third angle is $\frac{1}{5}$ larger than the second angle. Find the sizes of the angles of the triangle.

iv) Two airplanes flying in opposite directions pass each other. One plane is flying 300 miles per hour, and the second plane is flying 240 miles per hour. How long will they fly before they are 1200 miles apart ?

v) A river has a current of 2 miles per hour. How fast would a man have to row a boat in still water to take the boat 3 miles down stream and back again in two hours ?

vi) A pilot is to fly a place with a wind of 35 miles per hour for 240 miles. He plans to return immediately, flying against the wind. At what speed must he fly the plane in still air to make the entire trip in 2 hours ?

vii) A tank can be emptied by one pipe in 30 minutes and by another pipe in 25 minutes. How long will it take to drain the tank if both pipes are used ?

viii) How much cream containing 25% butter-fat must be added to 15 gallons of milk containing 2.6% butterfat to obtain a grade of milk that has 4% butterfat ?

GRAPHS

Objectives

1. To acquire knowledge of basic concepts, terms, relationships etc, used in the study of graphs.
2. To develop skill in drawing graphs.
3. To develop an ability to translate numerical data into graphical representation and vice versa.
4. To develop neatness in graphical work.
5. To recognise degree of accuracy in measurement.
6. To develop skill in solving problems with the help of graphs.
7. To develop an ability to estimate magnitudes.

Terms:

1. Axes of reference
2. Origin
3. Coordinates (abscissa and ordinate)
4. Scale
5. Line graph
6. Trends
7. Table
8. Units

Understandings

1. The meaning and use of terms listed above.
2. Interpretation of line-graphs.
3. Selection of the proper scale and units.
4. Definitions or meanings.
5. Principles.

Definitions or meanings:

1. Axes of references: One horizontal line and one vertical line are used in such a way that they resemble a number line. These two lines are the reference lines or the axes of reference of the graph.

2. Origin: The point at which the two of reference lines meet is called the origin.

3. Coordinates: The distance of a point from the vertical and horizental lines are known as the abscissa and the ordinate they together form the coordinates of the point.

4. Scale: Scale is the ratio giving the proportion of representative measurement to the actual measurement.

5. Table: The data arranged in a systematic form that facilities drawing a graph is called a table.

6-7. Line graph and trend: A line graph shows different quantities by means of a line or curve drawn on a squared paper. Changes in the direction of the line show quickly how the quantities vary and make it possible to see the trend of the information. The distance of the line from the base of the graph indicates the size of the quantities represented on the graph.

8. Unit: A quantify by reference to which other quantities are measured.

Principles:

1. A point can be uniquely placed if the distance from the two axes of reference is known.

2. Numerical data can be represented in terms of distances on a line with reference to a fixed point.

Skills:

1. Accurate plotting of points
2. Proper handling of ruler
3. Reading and drawing graphs
4. Recognising degree of accuracy in measurement.
5. Converting mathematical statements into graphs.

Activities:

1. Draw a line graph on the board and let the pupils suggest:

   a. Title to the graph
   b. Horizental axis.
   c. Vertical axis
   d. The scale used.

2. Have pupils bring to class graphs concerning population, taxation, income, expenditure, production, etc. found in newspapers and magazines for interpretation.

3. Ask the pupils to identify line graphs from various graphs drawn on black-board.

4. Make graphs of (i) heights of students of your class, (ii) daily temperature of your city found in newspaper.
5. Make a chart of new words learnt.
6. Make a bulletin board about various types of graphs.
7. Study books and encyclopedias for situations in which line graph can be used.
8. Write an essay on advantages and dis-advantages of graphs.
9. Solve a few problems on 'Time and Work' 'Work and Wages', 'Time and Distance' etc. by graphs.

RELATIONS, FUNCTIONS AND GRAPHS

Objectives

1. To acquire knowledge of basic concepts, terms, principles and relationships regarding relations and functions.
2. To develop skill in writing symbols for ordered pairs, finding the domain and range of a relation and function and drawing graphs.
3. To develop an ability to apply the knowledge of relations, functions and graphs to different problems.
4. To develop an ability to translate mathematical data given in one form into another form.
5. To develop interest in reading mathematical material.
6. To develop an ability to locate valid and reliable information in mathematics.
7. To develop appreciation of the language and history of mathematics and its contribution to life and to the development of the other subjects.

Previous Knowledge

(a)

- i) The idea of set
- ii) Ways of writing a set
- iii) Empty set
- iv) Subset of a set
- v) Real numbers
- vi) Union and intersection of sets
- vii) Complement of a set.

(b) Order

(c) Equations and inequalities

(d) One-to-one correspondence

(e) Variable

(f) Plotting points in the cartesian plane.

(g) Graphs of equations and inequalities.

How Introduced

In introduction the idea of "relation" is given by comparing the physical characteristics like height, weight, age, etc. For example, if a certain person 'a' is taller than 'b', then we can say that 'a' is in that relation with 'b'.

Terms

(a) Ordered pair
(b) Relation
(c) Binary relations
(d) Co-ordinates -- first and second
(e) Domain of a relation
(f) Range of a relation
(g) Graph of a relation
(h) Inverse of a relation
(i) Conjunction and disjunction
(j) Complement of the graph of a sentence
(k) Function
(l) Many-one-correspondence
(m) Inverse f a function
(n) Independent variable
(o) Dependent variable
(p) Graph of a function
(q) Periodic functions
(r) Even and odd functions
(s) Polynomial functions

Definitions

(1) By "ordered-pair" we mean that there is a first member and there is a second member, and their order makes a difference.

(2) A relation from a set A to a set B is defined to be any set of ordered pairs having first members from set A and second members from set B (A and B may be the same set).

(3) The relations which involve ordered pairs or which relate the pairs of elements of some set are called Binary Relations.

(4) The elements of an ordered pair are called "co-ordinates". We usually refer to the first element as the first co-ordinate and to the second element as the second co-ordinate.

(5) The set of all first co-ordinates of a relation is called its domain.

(6) The set of all second co-ordinates of a relation is called its range.

(7) A pictorial representation of a relation is called a graph of the relation.

(8) The inverse of a relation R is the relation obtained from R by interchanging the first and second co-ordinates in every pair.

(9) A sentence consisting of two (or more) sentences joined by the word "and" is called a conjunction.

(10) If two or more sentences are joined with the word "or", another sentence of a different kind results. Such a sentence is called a disjunctions

(11) The graph of the negation of a sentence is called the complement of the graph of the original sentence.

(12) A relation in which there is only one ordered pair for each member of the domain is called a function.

(13) A correspondence in which two or more ordered pairs having the same second co-ordinate but the different first co-ordinate is called a many-one-correspondence.

(14) If a variable depends on some other variable for its values (such as f (x) depends on x), the later is said to be the independent variable and the former as the dependent variable.

(15) A pictorial representation of a function is called the graph of the function.

(16) A function from R-R, where R is a set of real numbers, is called periodic if, and only if, $f(x)$ is not the same for all X and there is a real number p such that $f(x+p)=f(x)$ for all X in the domain of f. The smallest positive number for which this holds is called the period of the function.

(17) A function 'f' from R to R is said to be even if $f(x) = f(-x)$ for all x in the domain of f.

(18) A function 'f' from R to R is called odd if and only if $f(x) = -f(-x)$ for all x in the domain of f.

(19) A function of the type $f(x) = P(x)$ is a polynomial in x is called polynomial function.

Understandings and Principles

(1) Given a relation, one can construct a set of ordered pairs on the basis of it, or represent it by a set of ordered pairs.

Conversly, when a set of ordered pairs is given, the relation they represent is usually apparent, or can be found.

(2) When a relation is from a set A to a set B, its domain is some subset of A and its range is a subset of B.

(3) When the inverse of a relation is constructed the original range and domain are interchanged.

(4) The set of all points in a plane correspond to all the ordered pairs of real numbers.

(5) To make a graph of a relation from a set A to a set B, we draw some kind of picture showing all the possible ordered pairs with first co-ordinate from A and second co-ordinate from B. Then we show on that picture, by some kind of marks, which of the ordered pairs are in the felation.

(6) In the graph of a compound sentence, in order for a point to be in the graph, its co-ordinates must make both parts of the sentence true.

(7) If a sentence defines a relation, its negation also defines a relation.

(8) If the graph of a relation has two or more points on the same vertical line then it is not a function otherwise it is.

(9) In case of a function for every first co-ordinate in the domain of the relation there is one and only one second co-ordinate to be found in the range of the relation.

(10) Given any ordered pair in the function, if we increase the first co-ordinate by any integral multiple of a certain fixed number, and leave the 2nd co-ordinate unchanged, another ordered pair is obtained which is in the function. The smallest number for which a repetitive fixed circle can be found is called the period of that function.

Skills:

(1) In constructing a set of "ordered pairs" w.r.t.a relation from a given set.

(2) In constructing a "relation" from a set to some other set.

(3) In drawing the graph of a relation and its inverse.

(4) In finding the range and domain of a relation and its inverse.

(5) In drawing the graph of a conjunction and disjunction.

(6) Recognising relations and function.

(7) In drawing the graph of a function.

(8) Evaluating functions for particular value of the independent variable.

(9) In recognising periodic, even and odd functions.

Points of Emphasis in Relations Functions and Graphs

(1) Point out that a relation is a set of ordered pairs, whereas its domain and range are sets of individuals.

(2) Point out that when a relation is from a set A to a a set B, its domain is some subset of A. Its range is some subset of B.

(3) Have students make up sets of ordered pairs, not necessarily numbers, with a specific relation in mind. Then have other students try to guess what the rule is. However, you should not let students get the idea that whenever there is a relation some rule must be in some one's mind.

(4) Give students a simple graph in integers and then have them name the relation by the rule method.

(5) Stress that any set of points on a plane is the graph of some relation.

(6) Have students make up some graphs of their own to test other student's understanding. Students would try to determine whether the graphs given are graphs of functions or of relations which are not functions.

(7) Stress that a function is merely a special type of relation.

(8) Point out that there are many functions which are not describable by equations. Also there are many functions which are difficult or impossible to describe by tables; and there are functions which are difficult or impossible to describe by graphs. These are three common ways of describing functions.

(9) In connection with symbolism f(x); it is important that 'f' represents a function which is a set of ordered pairs, and that 'f(x)' represents a second co-ordinate in the function. Stress that this symbolism

does not mean f times x.

(10) Ask the students to note that the graph of a linear function is a line and the graph of a quadratic function is a parabola.

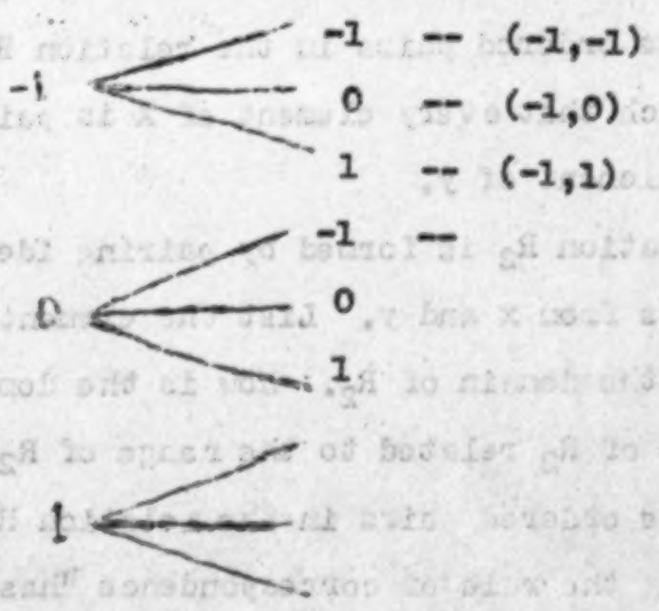
(11) Point out that the function value always refers to a second co-ordinate of a function. It never refers to the first co-ordinate of the function.

(12) Point out that a zero degree polynomial contains just one term without a variable. Any equation $y=k$, where k is a zero degree polynomial, has only one value, regardless of the first co-ordinate. Since the function value does not change as the first co-ordinate changes, we call the function a constant function.

(13) Some students may be able to predict the different appearance of graphs $y=ax^2$ as a varies. If so encourage them to do this. However encourage them to verify their predictions by actually plotting graphs.

Suggested Practice

1. Let U be the given set of numbers $\{-1,0,1\}$. If we wish to form all ordered pairs whose members belong to U, one way of doing it is illustrated in part below. You complete the 'tree'.



2. From the set $y = 32,16,8,4,2,$, a set of ordered pairs is specified by each of the following rules. Write symbols for each ordered pair in the set in every case.

a. The first number is a multiple of the second.

b. The first number is the square of the second.

c. The first number is the squareroot of the second.

d. The first number is two more than the second.

e. The first number is one-fourth of the second.

3. From the set $Z = \left\{ \pi\ \frac{22}{7}, 3.14159, 1\frac{10}{71} \right\}$, a set of ordered pairs is specified by each of the following rules. Write symbols for each ordered pair in the set in every case.

   a. The first number is to the right of the second on the number line.

   b. The first number is to the left of the second on the number line.

   Explain how the two sets of ordered pairs in a and b are related.

4. Given set $x = -1,0,1,2$ and set $y = -1,-1,0,1$.

   a. List the ordered pairs in the relation R1 from X to y such that every element of X is paired with every element of y.

   b. The relation R_2 is formed by pairing identical elements from x and y. List the elements of x which are in the domain of R_2. How is the domain of inverse of R_2 related to the range of R_2? Explain.

   c. List the ordered pairs in the relation R_3 from x to y by using the rule of correspondence "has the same absolute value as ". List the elements of x which are in the domain of R_3. List the element of y which are in the domain of R_3.

d. Find the ordered pairs in the inverse of R_3.

5. On a sheet of graph paper, draw and label a pair of co-ordinate axis, "first axis" and "second axis" and plot the following points:

a. (3,4)
b. (−5,2)
c. (0,0)
d. (−7,0)

e. (6,−7)
f. (−3,−9)
g. $(\frac{3}{4}, -\frac{2}{5})$
h. (− 2, 5)

6. Set U = −2,1,0,1,2 . The solution set of the sentence "y = x" can be denoted by (x,y) y = x . It is a set of ordered pairs, or a relation. List the elements in each of the following relations and tell the domain and range of each.

a. $\{(x,y) \mid x = y\}$
b. $\{(x,y) \mid x > y\}$
c. $\{(x,y) \mid x < y\}$

d. $\{(x,y) \mid x = y\}$
e. $\{(x,y) \mid x \neq y\}$
f. $\{(x,y) \mid x = y \text{ or } x > y\}$

7. The functions for real numbers listed below are defined by rules. In each exercise (i) plot some points whose co-ordinates satisfy the rules, (ii) state the domain of the function, (iii) state the range of the function, and (iv) if possible, write a formula which defines the function.

a. f = {(x,y) | the second co-ordinate is twice the first.}

b. q = {(x,y) | the second co-ordinate is two less than one half the first.}

c. h = {(m,n) | the second co-ordinate is four more than the square of the first}

d. m = {(e,f) | the second co-ordinate is four diminished by the square of the first}

8. a. Draw the graph of the function f whose domain is the set of integers $\{1,2,3,4,5\}$, whose range is the set of integers $\{4,5,6,7,\}$, and whose rule of correspondence is given by $f(1) = 5$, $f(2) = 4$, $f(3) = 4$, $f(4) = 6$ and $f(5) = 7$.

   b. What is the domain of the inverse of f?

   c. What is the range of the inverse of f?

   d. Draw the graph of the inverse of f and explain why the inverse of f is or is not a function.

9. a. Draw the graph of the function defined by $f(x) = x - (x)$ where the symbol $\lfloor x \rfloor$ denotes the greatest integer n such that $n \leq x$; for example, $\lfloor 3 \rfloor = 3$, $\lfloor 5.4 \rfloor = 5$, $\lfloor -3.2 \rfloor = -4$, $\lfloor \pi \rfloor = 3$, etc.

   b. Show that f is periodic with period 1.

10. State the constant of variation, describe the domain and range and state the rule as an open sentence, in the following:

    1. The circumference of a circle varies directly as a radius.

    2. Area of a rectangle with a constant height varies directly as the length of the base.

11. Which of the following graphs could be a rule for a function? Domain is $\{\ \}$; Range is $\{\ \}$

12. Suggest examples from Physics, Chemistry and other subjects, Possible areas are

    (i) Intensity of illumination varies inversely as the square of the distance from the source of light.

    (ii) Electrical resistance of a cable varies directly as its length and inversely as the square of its diameter.

    (iii) Boyle's Law.

LINE AND PLANE

Objectives

1. To acquire knowledge of basic concepts, terms, principles, and relationships in Lines and Planes.
2. To develop skill in using mathematical instruments in problems on Lines.
3. To develop the ability to apply the knowledge in different situations or problems.
4. To develop logical thinking and reasoning.
5. To develop the attitude of systematic presentation of arguments.
6. To develop an appreciation of form, figure, and design.

Terms

1. Space
2. Collinear set of points
3. Coplaner set of points
4. Convex set
5. Half-plane
6. Edge
7. Half-space

Understandings

Understanding that

1. Space is the set of the all points
2. a set of points is collinear if there is a line which contains all the points of the set.
3. a set of points is coplaner if there is a plane which contains all the points of the set.
4. for every two points there is exactly one line that cont both points.
5. every plane contains at least three non-collinear points.
6. space contains at least four non-coplaner points.
7. if two different lines intersect there intersection contains only one point
8. if two points of a line lie in a plane then the line lies in the same plane.
9. if a line intersects a plane not containing it, the intersection contains only one point.
10. given a line and a point not on the line, there is exactly one plane containing both
11. any three points lie in at least one plane, and any three noncollinear points lie in exactly one plane
12. given two intersecting lines, there is exactly one plane containing both.
13. if two planes intersect, there intersection is a line
14. a set A is called convex if for every two points P and Q of the set, the entire segment PQ lies in A.

15. given a line and a plane containing it. The points of the plane that do not lie on the line form the two sets such that

    a) each of the sets is convex
    b) if P is in one of the sets and Q is in the other, then the segment PQ intersects the line.

16. given a line L and a plane containing it, the two sets described in (14) are called half planes or sides of L, and L is called the edge of each of them

17. If two points lie in the same half-plane, then the segment joining them lies in the same half-plane and so never intersects the line.

18. if two points lie in the opposite half-planes, then the segment joining them always intersects the line

19. the points of space that do not lie in a given plane form two sets, such that

    a) each of the sets is covex, and
    b) if P is in one of the sets and Q is in the other, then the segment PQ intersects the plane.

20. the two sets described in (18) are called half-spaces, and the given plane is called the face of each of them

21. every plane in a space is a face of only two half-spaces.

Skills

1. To recognise concepts, terms etc. when illustrated.
2. To illustrate terms, concepts, etc.
3. To discriminate between closely related concepts, and principles e.g. collinear set and coplanar set, half plane and half space etc.
4. To define or describe terms etc. whenever possible
5. To detect errors in statements, definitions etc.
6. To compare and contrast terms, concepts and principles.
7. To read and draw geometric figures.
8. To represent a given statement in symbolic form and vice versa.
9. To analyse a data and find what is given and what is to be found.
10. To find the adequacy or otherwise of the data.

Suggested Activities

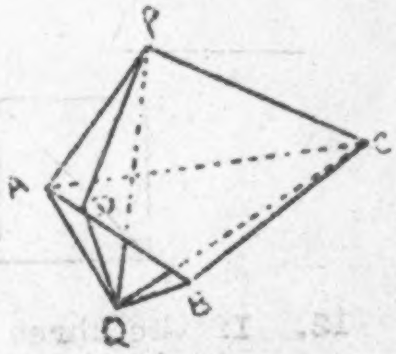
1. Hold your arm straight out in front of you consider a point A, at the tip of your fore finger, and a point B, at the upper right front corner of your room. How many lines contain both points A and B? What postulate supports your answer?

2. Take your book or a piece of stiff card board, Can you support it in a fixed position on the ends of two pencils. What is the minimum number of pencils needed to support in this manner?

3. Let a corner of your desk represent a point P, the light switch on the wall a point Q, and a corner of the room a point R. Is there a plane containing points P, Q, and R?

4. Hold two pencils together by their sharpened ends between your thumb and fore finger. If the pencils represent two intersecting lines, how many planes will contain both these lines?

5. Make a model like the adjacent figure (three diamensional) using toothpicks and glue. Decide by looking at this figure whether each of the following set of points is 1) collinear 2) not collinear but coplonar 3) not coplnonar.



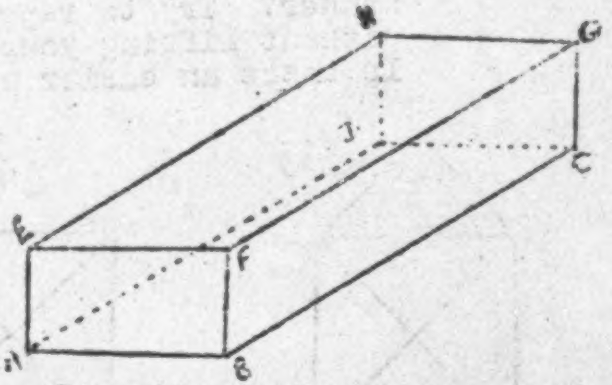
a) A, B, C, D

b) A, D, B

c) P, D, Q

d) P, B, C

e) A, B, C, Q



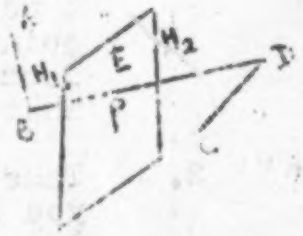
6. Examine this figure of a rectangular solid until you see how it is drawn to look like a three diamensional figure. Then close the book and draw a figure like this from the memory. Practice until you are satisfied with the results.

7. Sketch any geometrical solid bounded by plane surfaces such that the set of points in the interior of the figure is not convex.

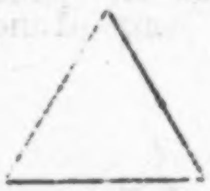
8. Draw a plane quadrilateral whose interior is convex. Draw one whose interior is not convex.

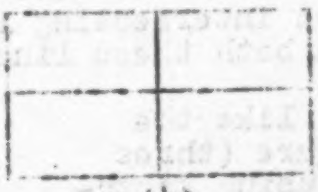
9. Draw two half-planes which have a common edge and are coplanar. Draw two which have a common edge but are not coplanar.

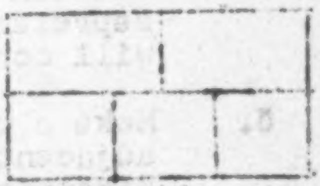
10. Complete the following by referring to the figure
In the figure, ________________ E separates
spaces into __________ H, and __________.
We know that A and __________ are
on the same side of __________ since
__________ does not intersect plane E
Also, B and D are on ______________
of E, since ____________________
____________________ by showing that
A and ____________________ are on __________
of plane E.

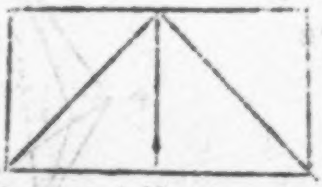


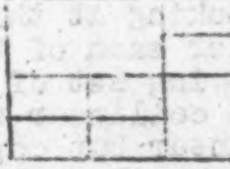
11. In this problem, you "win" if you can cross each segment of the figure exactly once without lifting your pencil from the paper. Copy the figures on a piece of paper and see whether you can discover in which two of the five figures it is possible for you to 'win'. Is there a way to make up figures for which you must always 'lose'?


(a)

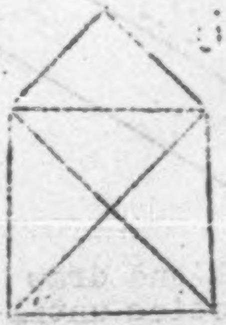

(b)

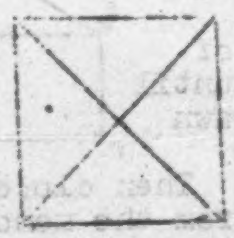

(c)

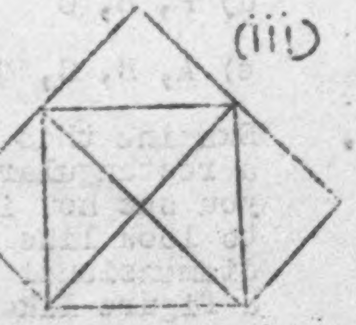

(d)


(e)

12. If the three figures shown, two can be drawn without lifting your pencil or retracing a line segment, while the third one cannot. Which two can be drawn in this manner? Try to reproduce each figure on your paper without lifting your pencil or retracing a segment. Is there an easier way of arriving at a conclusion.


(i)


(ii)


(iii)

13. Distinguish between the following sets of points:

(a) Straight line (b) Segment

14. Which of the following have mid points.

(a) Straight lines (b) rays (c) segments.

15. Display a number of optical illusions to show the danger of basing conclusion on appearance.

16. Discuss the necessity of having certain terms udefined.

17. Develop inductively an understanding of axioms and postulates as they are needed.

18. Ask pupils to explain why:-

On a level of floor, a four-legged table will some times rock, while a three legged table is always steady. Explain.

10. Ask pupils to mark the following statements true or false.

(a) The union of any two convex sets which have at-least two points in common is a convex set.

(b) If three points are collinear, then they are coplanar.

(c) If three points are coplanar, then they are collinear.

(d) Three dimensional space contains at least four points.

20. Ask pupils to explain.

Line L_1 intersects plane E in P, but does not lie in E. Line L_2 lies in plane E, but does not contain point P. Is it possible for L, to intersect L2?

21. Ask pupils to draw a line L separating the plane into two half planes. Label the half-planes H1 and H2. Pick points D and K in H1 and point F in H2.

(a) What is the intersection of $\overline{DK}$ and L? Why?

(b) What is the intersection of $\overline{KF}$ and L? Why?

ANGLES AND TRIANGLES

Objectives

1 To acquire the knowledge of basic concepts, terms, principles and relationships regarding angles and triangles.

2 To develop skill in using mathematical instruments in problems on angles and triangles.

3 To develop an ability to write mathematical language into concise form through the use of various symbols.

4 To develop the ability to apply knowledge regarding angle - types, and triangles in different situations or problems.

5 To develop logical thinking and reasoning.

6 To develop an attitude of systematic presentation of arguments.

7. To increase the pupils awareness and appreciation of the presence and usefulness of angles and triangles in nature, art, architecture, furniture, textile, etc.

Terms

1 Angle : side, vertex

2 Triangle

3 Interior of an angle

4 Exterior of an angle

5 Interior of a triangle

6 Exterior of a triangle

7 Initial side of an angle

8. Terminal side of an angle

9 Degree

10 Angular measure

11 Linear pair of angles

12 Supplementary angles: supplement of an angle

13 Right angle

14 Perpendicular

15 Complementary angles: complement of an angle

16 Acute angle

17 Obtuse angle

18 Congruent angles
19 Vertical angles.

Understandings

Understanding that:-

1. If two rays have the same end point, but do not lie on the same line, then their union is an angle, the two rays are called its sides and the end point its vertex.

2. If A, B, and C are any three-non-collinear points, then the union of segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ is called a triangle and is denoted by $\triangle$ ABC. The points A, B, and C are its vertices and the segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ its sides.

3. (A) a point P is in the interior of $\angle$ BAC if

    i) P and B are on the same side of the line $\overline{AC}$, and

    ii) P and C are on the same side of the line $\overline{AB}$

   (B) The exterior of $\angle$ BAC is the set of all points that lie neither on the angle nor in its interior.

4 a point lies in the interior of a triangle if it lies in the interior of each of the angles of the triangles. A point lies in the exterior of a triangle if it lies in the plane of the triangle but does not lie on the triangle or in the interior.

5 in $\angle$ CAB, $\overline{AC}$ is the initial side and $\overline{AB}$ is the terminal side while in $\angle$ BAC, AB is the initial side and $\overline{AC}$ is th terminal side.

6 the number of degrees in an angle is called its measure.

7 to every angle $\angle$ BAC there corresponds a real number between 0 and 180.

8. if $\overline{AB}$ is a ray on the edge of the half-plane H, then for every number r between 0 and 180 there is exactly one ray AP, with P in H such that m$\angle$ PAB=r.

9. if D is in the interior of $\angle$ BAC then m$\angle$ BAC = m$\angle$ BAD + m$\angle$DAC (where m$\angle$ BAD means measure of angle BAD).

10. if $\overline{AB}$ and $\overline{AD}$ are opposite rays, and AC is any other ray, then $\angle$ BAC and $\angle$ CAD form a linear pair.

11. if the sum of the measures of two angles is 180, then the angles are called supplementary, and each of them is called a supplement of the other.

12. if two angles form a linear pair, then they are supplementary.

13. if the angles in a linear pair have the same measure, then each of them is called a right angle.

14. a right angle is an angle whose measure is 90.

15. if $\overline{AB}$ and $\overline{AC}$ form a right angle, then they are called perpendicular.

16. if the sum of the measures of two angles is 90, then the angles are called complementary and each of them is called a complement of the other.

17. an angle with measure less than 90 is called acute.
18. an angle with measure greater than 90 is called obtuse.
19. two angles with the same measures are called congruent.
20. if two angles are complementary, then both are acute.
21. every angle is congruent to itself.
22. any two right angles are congruent.
23. if two angles are both congruent and supplementary, then each is a right angle.
24. supplements of congruent angles are congruent.
25. complements of congruent angles are congruent.
26. two angles are vertical angles if their sides from two pairs of opposite rays.
27. vertical angles are congruent.
28. if two intersecting lines form one right angle, then they form four right angles.

Skills

1 To illustrate terms, concepts etc. wherever required.

2 To define or describe concepts, terms etc, wherever possible.

3 To recognise concepts, terms etc. when illustrated.

4 To discriminate between closely related concepts and principles.

5 To identify correct statements regarding angles and triangles.

6 To analyse data and to find what is given and what is to be found.

7 To find the adequacy or otherwise of data.

8 To use geometrical instruments in drawing angles and triangles according to given data.

9 To represent given statements in symbolic form and vice versa.

10 To estimate the magnitude of an angle or the sides of a triangles.

Activities

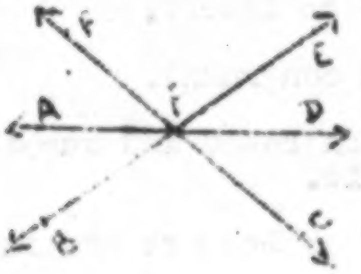
Ask pupils to complete the following definitions:

(a) An angle is the ______ of two ______ which have the same ______ but which are not ____________.

(b) A triangle is the ___________ of the three ____________ joining each pair of three ___________.

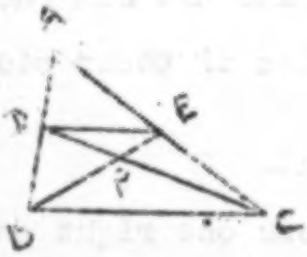
2. Let pupils make the following diagrams and ask:

(a)



How many angles are in this figure? (There are more than six.)

(b)



How many triangles are in this figure? (There are more than five.)

3 Make the following diagram and ask:

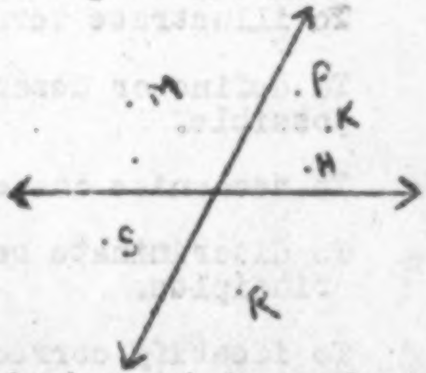
the pupils: Which points of the figure are in

(a) the interior of ∠ CBA?

(b) the exterior of ∠ EBC?

(c) the interior of ∠ ABD?

(d) the interior of ∠ ABQ?



4 Ask pupils to use a geometric instrument to evaluate each of the following:

(a) m ∠ APB

(b) m ∠ DPB

(c) m ∠ EPC

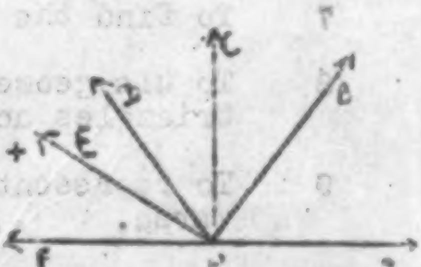
(d) m ∠ FPC

(e) m ∠ APC

(f) m ∠ EPB

(g) m ∠ BPF

(h) m ∠ FPD + m ∠ EPD.



5 Ask pupils to sketch angles whose measures are 30, 40, 60, 120 by using only a straightedge and not a protractor and check them by a protractor.

6 Ask pupils to write the following statements into symbolic form and vice versa:

(i) Line segments AB and CD are perpendicular.

(ii) Angles ABC and DEF are congruent.

(iii) $m \angle A \cong m \angle A$

(iv) The sum of the measures of the angles of a triangle ABC is the straight angle.

(v) $m \angle BAC + m \angle CAD = 90$

7 Ask pupils to identify the hypothesis and the conclusion for each of the following statements:

(i) If two angles are complementary, then each of them is acute.

(ii) If two angles are both congruent and supplementary, then each of them is a right angle

8 Ask pupils to write each of the following statements as an "If ....then" statement.

(i) Supplements of congruent angles are congruent.

(ii) Two angles which form a linear pair are supplementary.

9 Take a trip through the school building to see how geometry plays a part in the stairways, hall and general architecture of the building.

10 Collect and display angle measuring instruments, such as the transit, magnetic compass, field protractor, carpenter's square etc. Make a field protractor and use it out of door.

11 Ask pupils for common examples of perpendicular lines, such as:

i) Adjacent sides of most sheets of writing paper.

ii) Adjacent sides of most window frames.

iii) Intersecting sides of rectangles and squares.

iv) The bottom and any side of most boxes.

12 Represent a triangle on paper, shading the three angles. Cut out the figure. Tear off the angles and add them by placing the vertices together and the sides adjacent. The three angles form a straight angle.

13 Prepare a variety of carboard models of triangles and number them. Use these for a lesson on classification of triangles rather than the ones in the book. Move them about, changing the base.

14 Give the pupils three angles totaling 180° and ask them to make a triangle which will include the three angles. There will be as many sizes of triangles as there are numbers of the student, but all will have the same shape. Pupils should discover from this the necessary condition for similar triangles.

CONGRUENCE

Objectives

1. To acquire knowledge of basic concepts, terms, principles and relationships regarding congruence.

2. To develop skill in using mathematical instruments in problems on congruence.

3. To develop an ability to apply the knowledge of congruence to different situations and problems.

4. To develop logical thinking and reasoning.

5. To develop an attitude of systematic presentation of arguments.

6. To increase the pupils' awareness and appreciation of the presence and usefulness of congruence in different forms and figures in nature, art, architecture.

Terms

1. one-to-one correspondence
2. Congruence
3. Identity congruence
4. Side included by angles
5. Angle included by sides
6. Shape
7. Size
8. Similar (triangles)
9. Bisector of an angle
10. Equilateral triangle
11. Isoscales triangle
12. Scalene triangle
13. Base
14. Base angle
15. Vertex angle
16. Equiangular
17. Quadrilateral: rectangle, square
18. Median

Understandings

Understanding that:

1. one-to-one correspondence between the vertices of the two triangles is a matching-up scheme.

2. if the matching-up scheme can be made to work, that is, if the triangles can be made to fit when the vertices are matched up in the precribed way, then the one-to-one correspondence is called a congruence between the two triangles.

3. in $ABC \leftrightarrow BED$ the first letter on the left corresponds to the first letter on the right, the second corresponds to the second and the third corresponds to the third

4. the correspondence $ABC \leftrightarrow ABC$ is called the identity congruence

5. angles are congruent if they have the same measure. Segments are congrument if they have the same length.

6. every segment is congrument to itself.

7. given a correspondence $ABC \leftrightarrow DEF$ between the vertices of two triangles. If every pair of corresponding sides are congruent, and every pair of corresponding angles are congruent, then the correspondence $ABC \leftrightarrow DEF$ is called a congruence between the two triangles.

8. a side of a triangle is said to be included by the angles whose vertices are the end points of the segment.

9. an angle in a triangle is said to be included by the sides of the triangle which lie in the sides of the angle

10. every SAS correspondence is a congruence

11. every ASA correspondence is a congruence

12. every SSS correspondence is a congruence

13. every angle has exactly one bisector

14. if two sides of a triangle are congruent, then the angles opposite these sides are congruent.

15. a triangle with two congruent sides is called isoscales. The remaining side is the base. The two angle that include the base are base angles. The angle opposite the base is the vertex angle.

16. a triangle whose three sides are congruent is called equilateral

17. a triangle no two of whose sides are congruent is called scalene.

18. a triangle is equiangular if all three of its angles are congruent.

19. every equalateral triangle is equiangular

20. if two angles of a triangles are congruent, then the side opposite them are congruent.

21. every equiangular triangle is equilateral

22. Let A, B, C and D be four coplanar points, if no three of these points are collinear, and the segments AB, BC, CD, and DA intersect only at their end points, then the union of the four segments is called a quadrilateral

23. if all four angles of a quadrilateral are right angles, then the quadrilateral is a rectangle

24. if all four of angles are right angles and all four sides are congruent, then the quadrilateral is a square

25. a median of a triangle, is a segment whose end points are a vertex of the triangle and the mid point of the opposite side

26. every triangle has three medians - one for each vertex.

27. a segment is an angle bisector of a triangle if

    i) it lies in the ray which bisects an angle of the triangle, and

    ii) its end points are the vertex of this angle and a point of the opposite side.

Skills

1. To illustrate terms, concepts etc. whenever required.

2. To define or describe terms regarding congruence.

3. To recognise congruent geometric figures.

4. To draw figures congruent to given figures.

5. To analyse data and find what is given and what is to be proved.

6. To present systematically the arguments in proving theorems and statements.

7. To select a congruence postulate most suitable in proving a theorem or a statement

8. To find the adequacy or otherwise of data

9. To discriminate between closely related concepts such as equiangular and equilateral, one-to-one correspondence and congruence, congruence and similarly, median and angle bis-ector, shape and size etc.

Activities

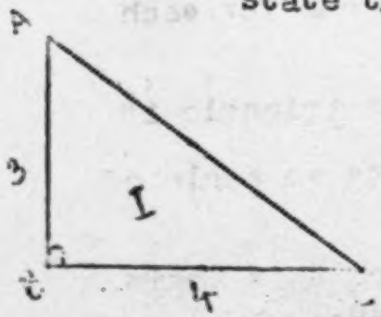
1. Point out that congruent figures are figures which have the same size and same shape; they are exact duplicates of each other.

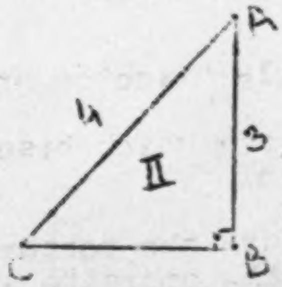
2. Take two leaves having the same size and same shape and make them coincide so that their corresponding parts fit together.

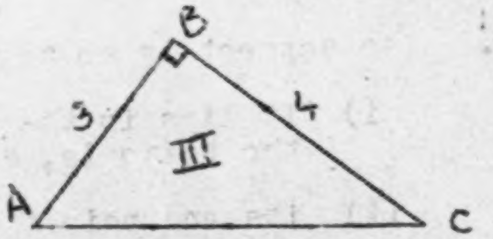
3. Point out that the aim of mass production is to manufacture exact duplicates or congruent figures.

4. Note, in the congruent triangles, how corresponding parts may be located. Corresponding sides lie opposite equal angles and corresponding angles lie opposite equal sides.

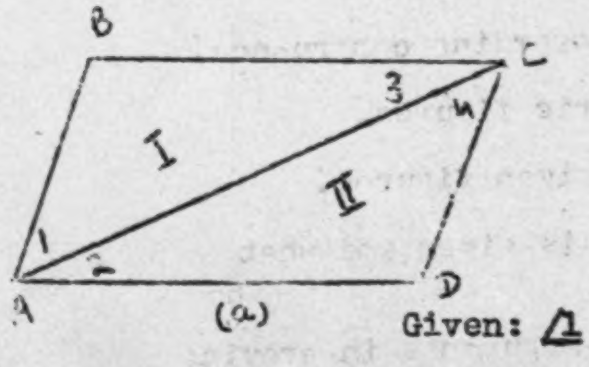
5. Make the following triangles on the board and ask the pupils to select congruent triangles and to state the congruency principles.

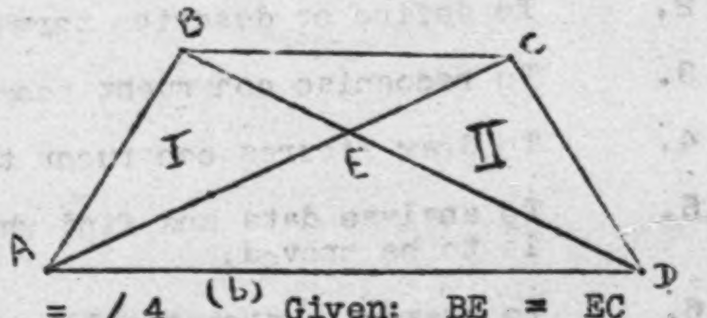






6. In each of the following $\triangle$ I can be proved congruent to $\triangle$ II. Make a diagramme showing the equal parts of both triangles and state the congruency principles.





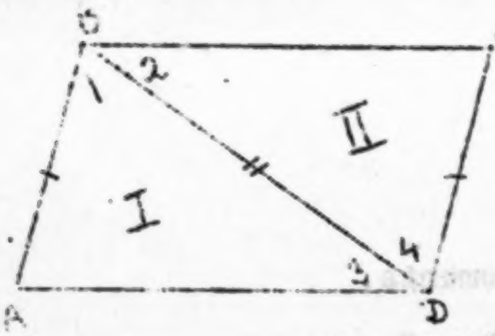
Given: $\angle 1 = \angle 4$ Given: BE = EC

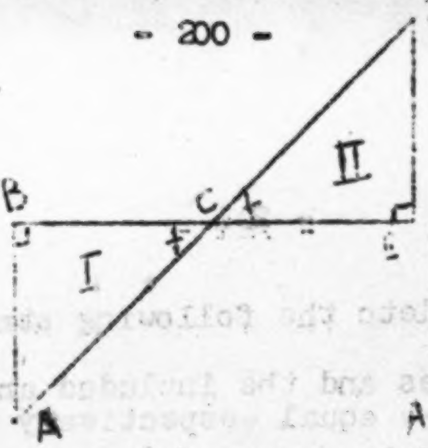
$\angle 2 = \angle 2$ AE = ED

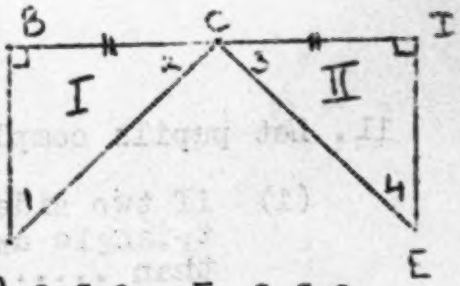
To prove: $\triangle 1 = \triangle II$ To prove: $\triangle I = \triangle II$

7. Make the following figures and ask the students to find additional parts needed to prove:

$\triangle I = \triangle II$


B
C
1
2
II
I
3
4
A
D

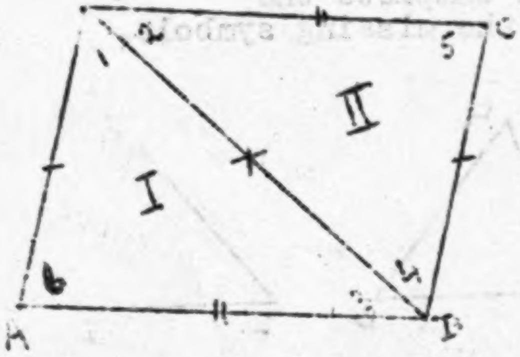


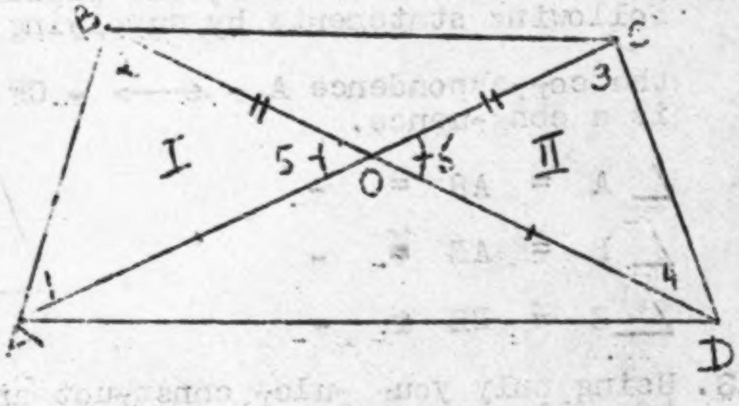

B
C
I
2
3
I
II
1
4
A
E

(a) By s.s.s. = s.s.s. (c) a.s.a. = asa (a) a.s.a. = a.s.a.

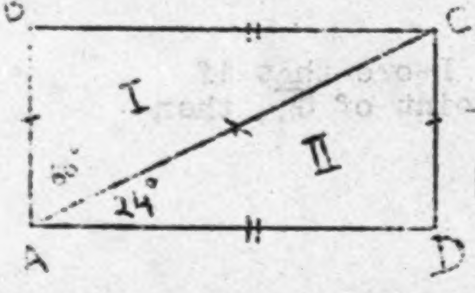
(b) By s.s.s. = s.s.s. (b) s.s.s. = s.a.s.

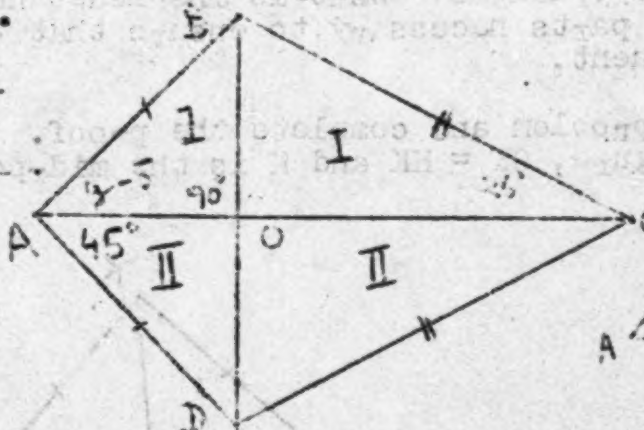
8. Draw the following figures and ask the students to select corresponding parts when in each the equal parts needed to prove ∠ I = ∠ II are marked.

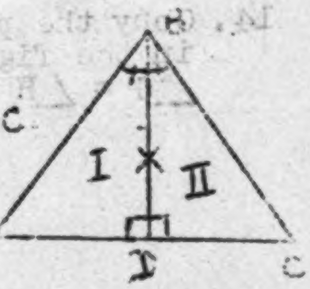

B
C
1
2
5
II
I
6
4
3
A
D


B
C
2
3
I
5
O
6
II
1
4
A
D

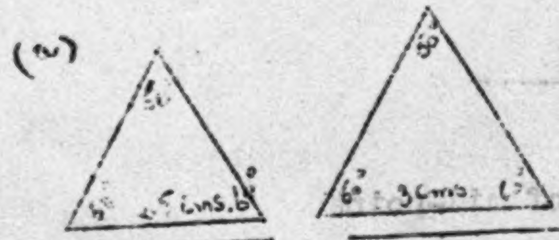
9. Show that Algebra can be applied to equal corresponding parts of congruent triangles. In other words use the following kind of figures and find the values of x and y.

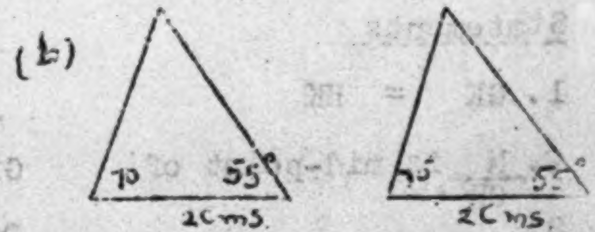


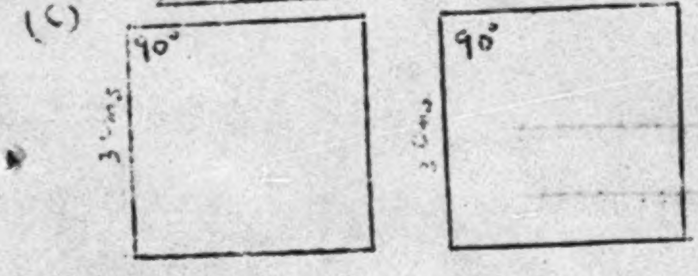

B
I
I
90°
A
45°
O
C
II
II
D

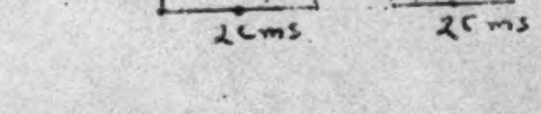


10. Let pupils suggest which of the following pairs of figures are congruent.


(a)
60°
60°
5cms.
60°
60°
3cms
60°


(b)
70
55
2cms.
55
2cms


(c)
90°
3 cms
90°
3 cms



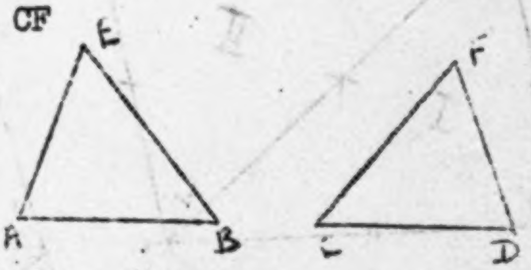
11. Let pupils complete the following statements.

(i) If two sides and the included angle of one triangle are equal respectively to .... than ....

(ii) If the hypotenuse and a leg of one right triangle are equal, respectively, to the............ of another right triangle, then the triangles are ....

12. If ABE = DCF, let pupils complete the following statements by supplying the missing symbols.

the correspondence A - ⟷ - CF is a congruence.



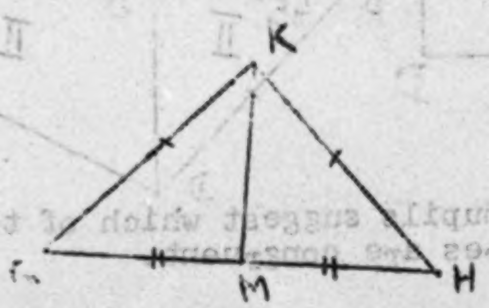
∠A ≅ AB ≅ -

∠B ≅ AE ≅ -

∠E ≅ BE ≅ -

13. Using only your ruler construct any triangle which has no two sides congruent. Next construct a second triangle congruent to the first triangle and describe the steps that you take. Is there more than one way of obtaining the second triangle from the first? How many of the six parts of the first triangle did you use in forming the second triangle? What is the least number of congruent parts necessary to ensure that the two triangles are congruent.

14. Copy the problem and complete the proof. Prove that if in the figure, GK = HK and M is the mid-point of $\overline{GR}$, then ∠G = ∠H



Proof

Statements	Reasons
1. GK = HK	________
2. M is mid-point of GH	Given
3. ________	Definition of mid-point
4. ________	Identity
5. △GMK ≅ △HMK	________
6. ________	________

15. Are the following statements true or false? Explain your answers.

    (a) The bisector of an angle lies entirely in the interior of the angle.

    (b) The bisector of an angle forms two acute angles with the sides of the angle.

16. Select the choice which correctly completes each sentence.

    (a) An angle bisector is a

    (i) segment (ii) ray (iii) plane

    (b) An equilateral triangle is

    (i) isosceles (ii) Scalence (iii) not isoscales

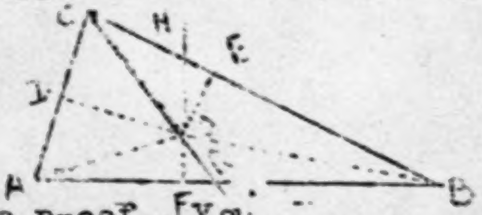
    (c) A corollary is a

    (i) definition (ii) postulate (iii) theorem

    (d) If two angles of a triangle are congruent we can conclude that it has two congruent sides according to a

    (i) definition (ii) corollary (iii) theorem

17. Find the fallacy in this argument that any triangle isososceles



Given: Any $\triangle$ ABC

Prove: $\triangle$ ABC is isosceles

Here is the outline of the proof. You complete it and write the authorities for the statements.

1. Let CR bisect $\angle$ C

2. Let GH be the perpendicular bisector of AB intersecting AB at F.

3. 3. Let O be the intersection of CR and GH.

4. Let CD be the perpendicular from O to BC

5. Draw OA and CB

6. Draw $\triangle$ ODC $\cong$ $\triangle$ OFC

7. CD = CE

8. $\triangle$ AFO $\cong$ $\triangle$ FBC

9. $\triangle$ ADO $\cong$ $\triangle$ BEC

10. AD = BE

11. AC = BC

12. $\triangle$ ABC is isosceles

Draw, now, the figure using proper geometric instruments and see what happens.

18. Let pupils suggest congruency used in clothing (sleeves, pockets, pant legs), industry and design.

19. Have pupils construct triangles when SSS; SAS; or ASA are given. Then construct three triangles congruent to them cut out and place one on the other to illustrate the three congruent postulates.

CIRCLE

Objectives

1. To acquire knowledge of basic concepts, terms, principles and relationships in circles.

2. To develop skills in using mathematical instruments in problems of circles.

3. To develop the ability to apply the knowledge in respect of circle in different situations and problems.

4. To develop the ability to build up a logical sequence of steps in proving statements regarding circles

5. To develop the attitude of systematic presentation of arguments.

6. To increase the pupils awareness and appreciation of the presence and usefulness of geometric forms in nature, art, architecture furniture, textile, etc.

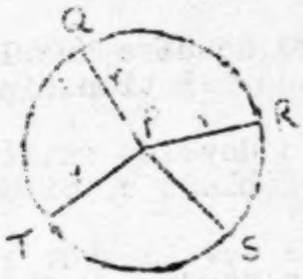
Terms

1. Circle - centre, radius
2. Concentric circles
3. Chord of a circle
4. Secant of a circle
5. Interior and exterior of a circle
6. Tangent line to a circle - a point of contact.
7. Tangent circles - internally and externally tangent circles.
8 Congruent circles.
9. Central angle of a circle.
10. Arc of a circle - major arc and minor arc
11. Degree measure of an arc.
12. Inscribed angle in an arc
13. Arc intersepted in an angle.
14. Inscribed and circumscribed quadrilaterals
15. Congruent arc
16. Secant segment to a circle
17. Tangent segmnt to a circle
18. Power of a point w.r.t. a circle

Understandings

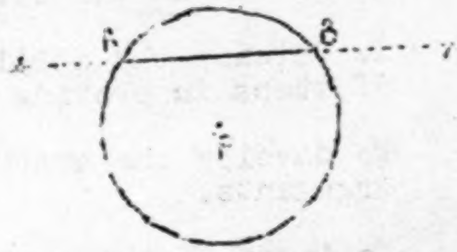
Understanding that:

1. A circle with centre P and radius r is the set of all points of the plane whose distance from P is equal to r.



2. Two or more circles with the same centre are said to be concentric.

3. A chord of a circle is a segement whose end points lie on the circle.



4. A line which intersects the circle in two points is called a secant of the circle.

5. An interior of a circle is the set of all points plane whose distances from the centre are less than the radius and the exterior of a circle is the set of all points whose distances from the centre are greater than the radius.

6. A tangent to a circle is a line which intersects the circle in one and only one point. This point is called the point of tangency or point of contact.

7. Every circle has a tangent at each of its points.

8. Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

9. Two circles are tangent if they are tangent to the same line at the same point. If the two tangent circles are coplaner, and their centres lie on the same side of their common tangent, then they are called internally tangent, but if their centres are on opposite sides of their common tangent, then they are called externally tangent.

10. The perpendicular from the centre of a circle to a chord bisects the chord.

11. The segment from the centre of a circle to the mid-point of a chord is perpendicular to the chord.

12. No circle contains three collinear points.

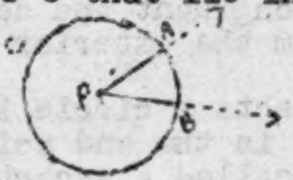
13. Circles with congruent radii are called congruent circles.

14. In the same or in congruent cirles, chords equidistant from the centre are congruent and conversely the two congruent chords are equidistant from the centre.

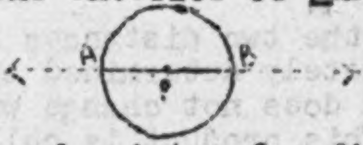
15. If a line intersects the intereior of a circle, then it intersects the circle in two and only two points.

16. A central angle of a circle is an angle whose vertex is the centre of the circle.

17. The minor arc AB of a circle is the union of A, B and all points of C that lie in the interior of ∠APB



18. The major arc AB is the union of A, B, and all points of C that lie in the exterior of ∠APB



19. If A and B are the end points of a diameter, then we get two arcs each of which is called a semicircle

20. The degree measure of an arc is the measure of the corresponding central angle.

21. An angle is inscribed in an arc, if

    a) The sides of the angle contain the end points of the arc and

    b) The vertex of the angle is a point, but not an end point, of the arc.

22. An angle intercepts an arc if

    a) the end points of the arc lie on the angle

    b) all other points of the arc are in the interior of the angle, and

    c) each side of the angle contains an end point of the arc.

23. The measure of an inscribed angle is half the measure of its intercepted arc.

24. An angle inscribed in a semicircle is a right angle

25. Every two angles inscribed in the same arc are congruent.

26. A quadrilateral is inscribed in a circle if the vertices of the quadrilateral lie on the circle. If each side of the quadrilateral is tangent to the circle then the quadrilateral is circumscribed about the circle.

27. In the same or congruent circles, two arcs are called congruent if they have the same measure.

28. In the same or congruent circles, if two chords are congruent, then so are the corresponding arcs and its converse.

29. If $\overline{QA}$ is tangent to a circle at A, then $\overline{QA}$ is called a tangent from Q to the circle.

30 The two tangent segments to a circle from an exterior point are congruent and determine congruent angles with the segment from the exterior point to the centre.

31 If a segement intersects a circle in two points, and exactly one of these is the end point of the segment, then the segment is called a secant segment to the circle.

32. The product of the two distances from a point to the circle is completely determined by the given circle and that point, and does not change when we choose different secant lines. This product is called the power of that point w.r.t. the circle.

Skills

1. To recognise terms, concepts related to circles.

2. To illustrate terms, concepts, etc. of circles.

3. To discriminate between closely related concepts and principles, e.g. tangent and tangent segments, secant and secant segments, etc.

4. To detect errors in statements and definitions regarding circles and their allied concepts.

5. To define or describe terms whenever possible.

6. To compare and constrast terms, concepts and principles.

7. To transform verbal statements into geometrical figures and vice-versa.

8. To draw geometric figures with the given data using appropriate instruments.

9. To compute different parts of the circle from the given data.

10. To analyse the given data and to find what is given and what is to be foundout.

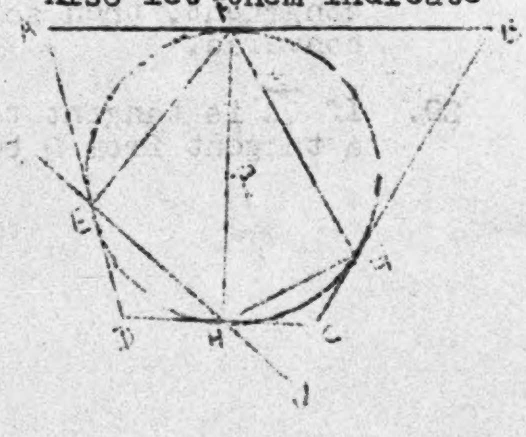
11. To find the adequacy or otherwise of the data.

12. To build up a logical sequence of steps in proving statements regarding circles.

Activities

1. Let the children collect the pictures of circular objects and fix them in their note books. Also let them indicate circular objects in classroom.

2. Draw a diagram, as drawn here, and match ea-ch figure on the left with one of the names on the right.



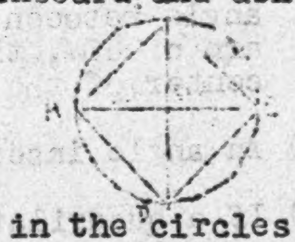
(a)	OE	1	Radius
(b)	FG	2	Central Angle
(c)	FH	3	Semicircle
(d)	CD	4	Minor Arc
(e)	IJ	5	Major Arc
(f)	EF	6	Chord
(g)	FGH	7	Diameter
(h)	FEG	8	Secant line
(i)	∠EOF	9	Tangent segement
(j)	Circle O about EFGH	10	Inscribed quadrilateral
(k)	Circle O in AB CD	11	Circumscribed quadrilateral
(l)	Quadrilateral EFGH	12	Inscribed circle
(m)	Quadrilateral ABCD	13	Circumscribed circle

3. Ask the students to draw several circles using the same centre and having different radii. Then ask the students:

   What kind of circles are these?
   Can you name a sport in which such circles are used?

   Explain them . . answers to the above questions in case they are not able to answer them.

4. Make the following diagram on the blackboard and ask the following questions:



   (a) What kind of triangle is OCD?
   (b) What kind of quadrilaterial is ABCD?.....

5. Let the students make various designs in the circles with hexagon, octagon, etc. as basis.

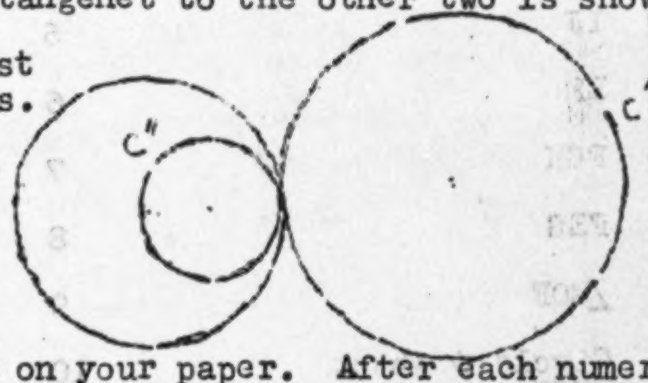
6. Ask the students to draw a rectangle on their note books and to inscribe a circle in the rectangle. Ask them the following questions:

   Can you inscribe a circle in the given rectangle? In case they say 'no', explain them the reason for that and further explain them that square is the only rectangle in which we can inscribe a circle.

7. Give the students sufficient practices in drawing tangents to different types of circles from points on the circle and from points outside the circle.

8 Some practice should be given in drawing central angles. Show the students that any radius in the circle may be used as the fixed line, and that with the center of the circle as the vertex any other line can be taken to form a central angle.

9 One arrangement of three circles having different radii so that each circle is tangenet to the other two is shown in this figure. Make sketches showing at least three other arrangements.



10. Write the numerals 1- 4 on your paper. After each numeral write 'Always' if the corresponding statement is alwasys true; write 'Never' if the statement is never true; write 'Sometimes' if the statement is sometimes true.

(a) The mid point of a circular arc and the centre of a circular are the same point.

(b) If a tangenet makes an angle of 30° with a chord at the point of contact, the chord is equal to a radius.

(c) If two chords of a circle bisect each other, both chords are diameters.

(d) If two equal chords intersect within a circle, the radius through the point of intersection bisects one of the pairs of vertical angles formed by the chords.

11. Write the numerals 1-3 on your paper. After each numeral write the expression that correctly completes the corresponding statement.

(1) In a circle whose radius is 6 cms., if an inscribed angle between a diameter and chord intersepts an arc of 60°, the chord is.......cms. from the center of the circle.

(2) An angle inscribed in a semi-circle is..........

(3) If a quadrilateral is inscribed in a circle, its opposite angles are............

12. Draw a circle with centre P and radius PQ = 1.5 cms. Locate a point A such that PA = 2 cms., and a point B such that PB = 1 cm., Now complete the following statements:

(a) A lies in the____of the circle because_________.

(b) B lies in the_____of the circle because_________.

(c) The circles with radii PA, PQ and PB are called

13 For each part of this problem answer in the following way.

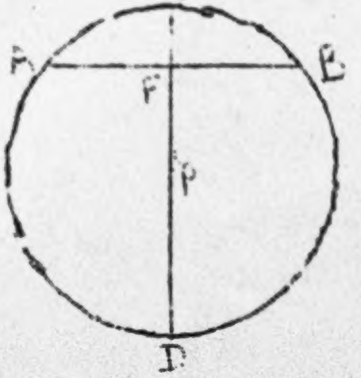
Write 'extra' if more information is given than is needed to get a numerical answer. Write 'not enough' if not enough information is given. Write 'O.K.' if just enough information is given to allow a numerical solution. Write 'contradictory' if the given infromation is contradictory.

Note. You do not need to solve; just decide whether or not you can.

In the figure P is the centre of the circle and $\overline{AB} \perp \overline{CD}$

(a) AF = 5, AB = ? (b) PB = 7, CD = ? (c) AC = 9, PB = ?

(d) CF = 3, EP = 2, PD = 6, CD = ?



14. Discuss the statement; 'Particles of mud fly off at a tangent from a revolving wheel'.

15. Use an indirect method to prove the following.

In the same circle or in equal circles, unequal chords have unequal arcs.

16. Study the figure of earth shadow. What should we know to understand the formation of these shadows.

17. A carpenter sometimes uses his steel square to determine the accuracy of a semicircular arc. Ask pupils to explain the principle involved.

18. Ask pupils to calculate the radius of a circular arch of a bridge which is 60 ft. wide and 14 ft. high.

THE STAFF OF THE NIH-HEW PROJECT NO.009.

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